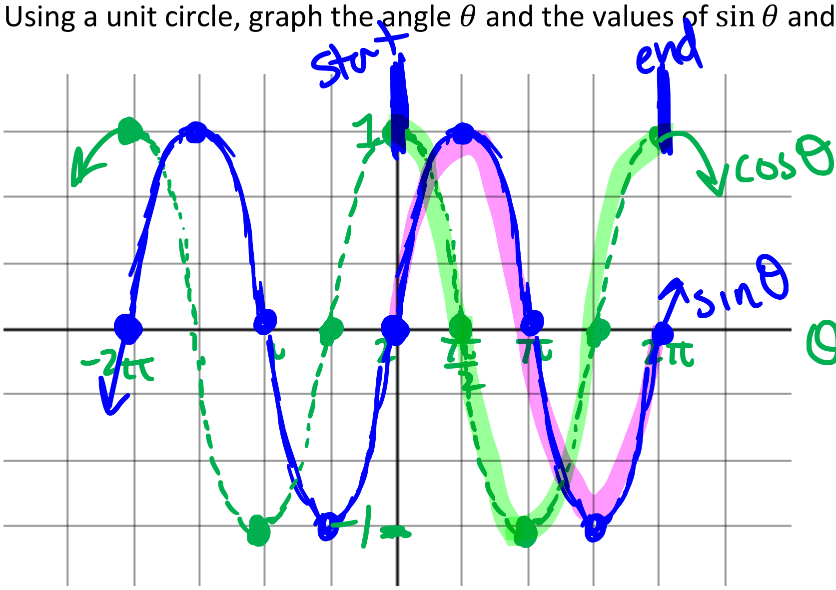
Graphing Sine and Cosine

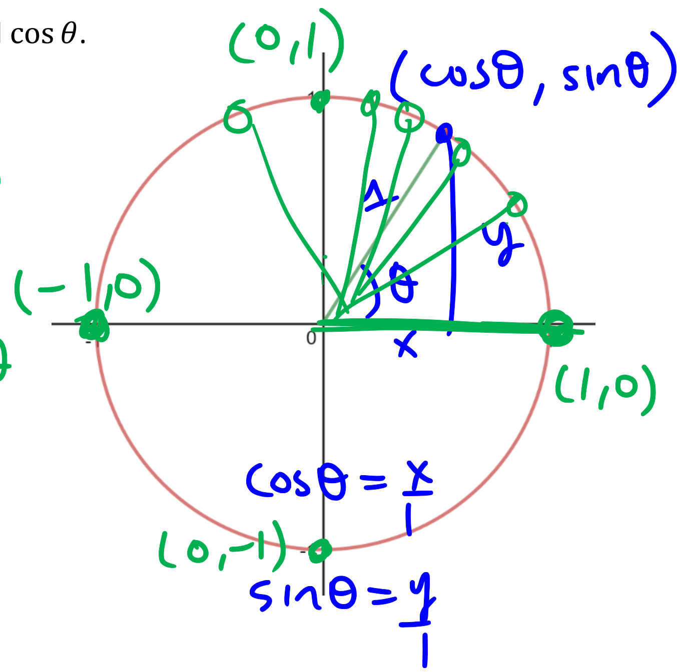
KNOW	DO	UNDERSTAND
How to identify the amplitude and period of a trig function. What a sinusoidal function looks like.	Can graph a trig function from the equation or characteristics accurately. Can build the equation of a trig function from the graph or characteristics accurately.	*Transformation:* Can explain how certain characteristics are or are not affected by a transformation. *Function Characteristics:* How the amplitude relates to the max/min values, midline as the average, period as the frequency, and shift as the start.

Vocab & Notation

- Amplitude
- Period
- Midline
- Phase Shift
- Sinusoidal function

Using a unit circle, graph the angle θ and the values of $\sin\theta$ and $\cos\theta$.





θ	0	$\pi/2$	π	$3\pi/2$	2π
$\cos\theta$	1	0	-1	0	
$\sin\theta$	0	1	0	-1	

Definition: Functions that repeat after a certain amount of time are called **periodic functions** (periodic meaning occurring at regular intervals). Periodic functions that have this regular "wave" shape are called **sinusoidal functions**.

We want to analyze this curve so that we can graph functions of the form: $f(x) = \sin x$

vert stretch (ROX) $\leftarrow$ $a \cdot \sin(b(x - c)) + d$ $\rightarrow$ shift up/down

horiz. stretch $\frac{1}{b}$ (ROY)

left/right shift

Definition: The **midline** is the average value of the function.

$f(x) = \sin x$, midline is $y = 0$

☆ only changed by vert. shift

Definition: The **amplitude** is the distance from the midline to the maximum or minimum, or equivalently, half the distance between the max and min.

$f(x) = \sin x$, amp is 1

☆ amplitude is always > 0 ☆

☆ only changed by vert. stretch ⇒ $|a|$

Definition: The **period** is the length of one complete cycle of a periodic function. Not necessarily how long it takes to repeat itself, but how long it takes to repeat the pattern.

$f(x) = \sin x$, period $= T = 2\pi$ (rad)

☆ horiz stretch changes period

☆ period is always > 0 ☆

$g(x) = \sin(bx) \Rightarrow T_{new} = \frac{2\pi}{b}$

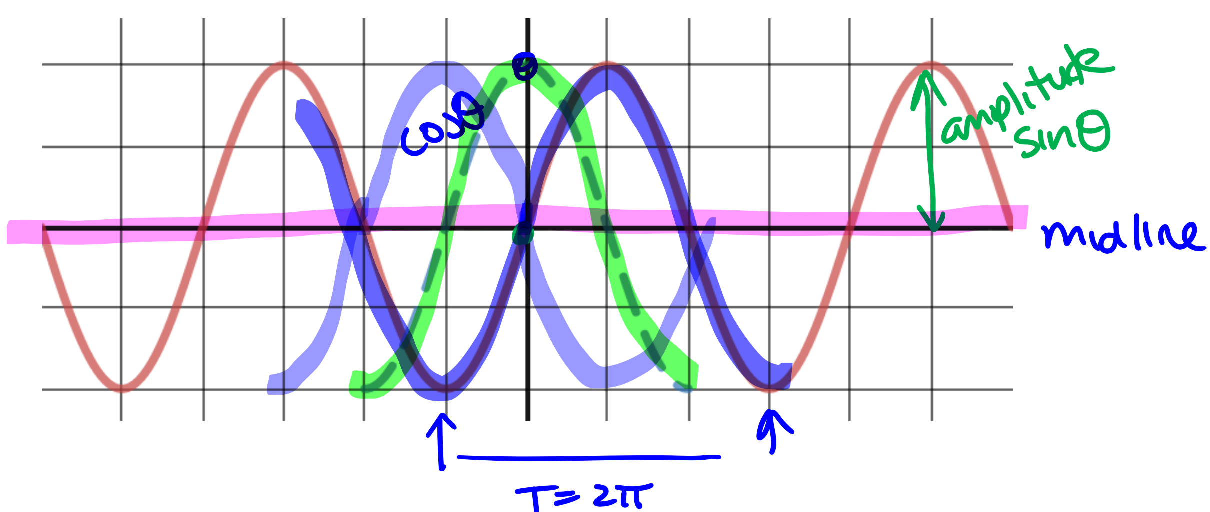
Definition: The **phase shift** is where the starting point of $\theta = 0$ got moved to.

$\sin x$ starts @ $y = 0$ $(0,0)$

$\cos x$ starts @ $y = 1$ $(0,1)$

☆ starting angle changes with a shift left/right.

When transforming a new function, we need to understand the basic function well to start.



$\cos(-\theta) = \cos\theta$

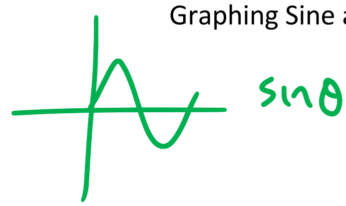
$\sin(-\theta) = -\sin\theta$

☆ cosine is even

☆ sine is odd

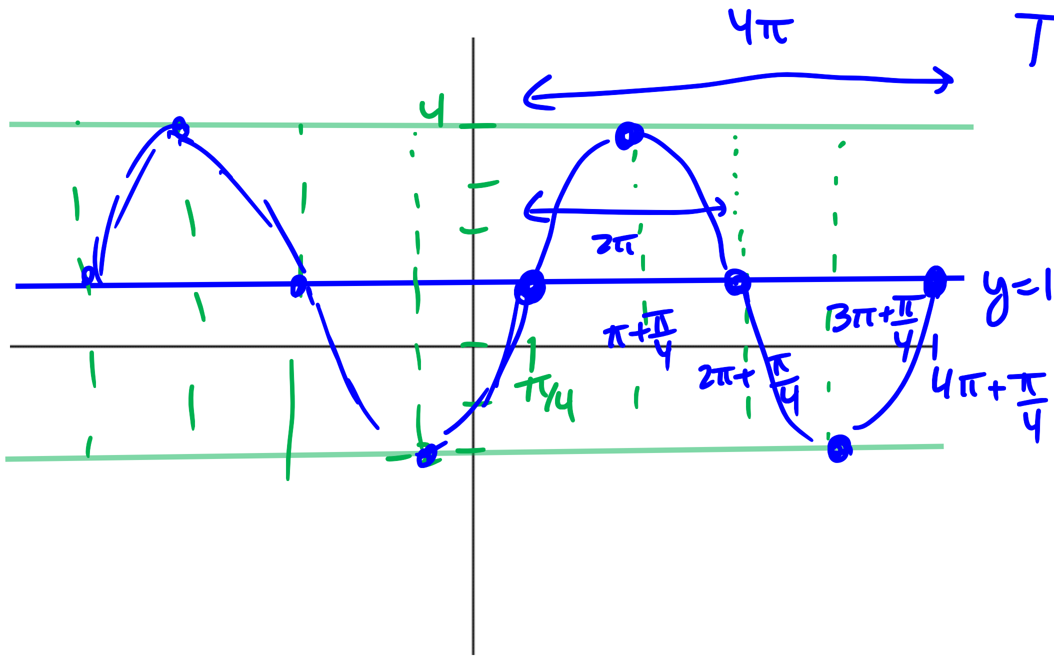
Example: Graph $f(\theta) = 3\sin\left(\frac{1}{2}\left(\theta - \frac{\pi}{4}\right)\right) + 1$

- Identify the midline from the vertical displacement
- Use the amplitude to find the max and min lines
- Use the phase shift to identify the starting point
- Split the period into quarters.



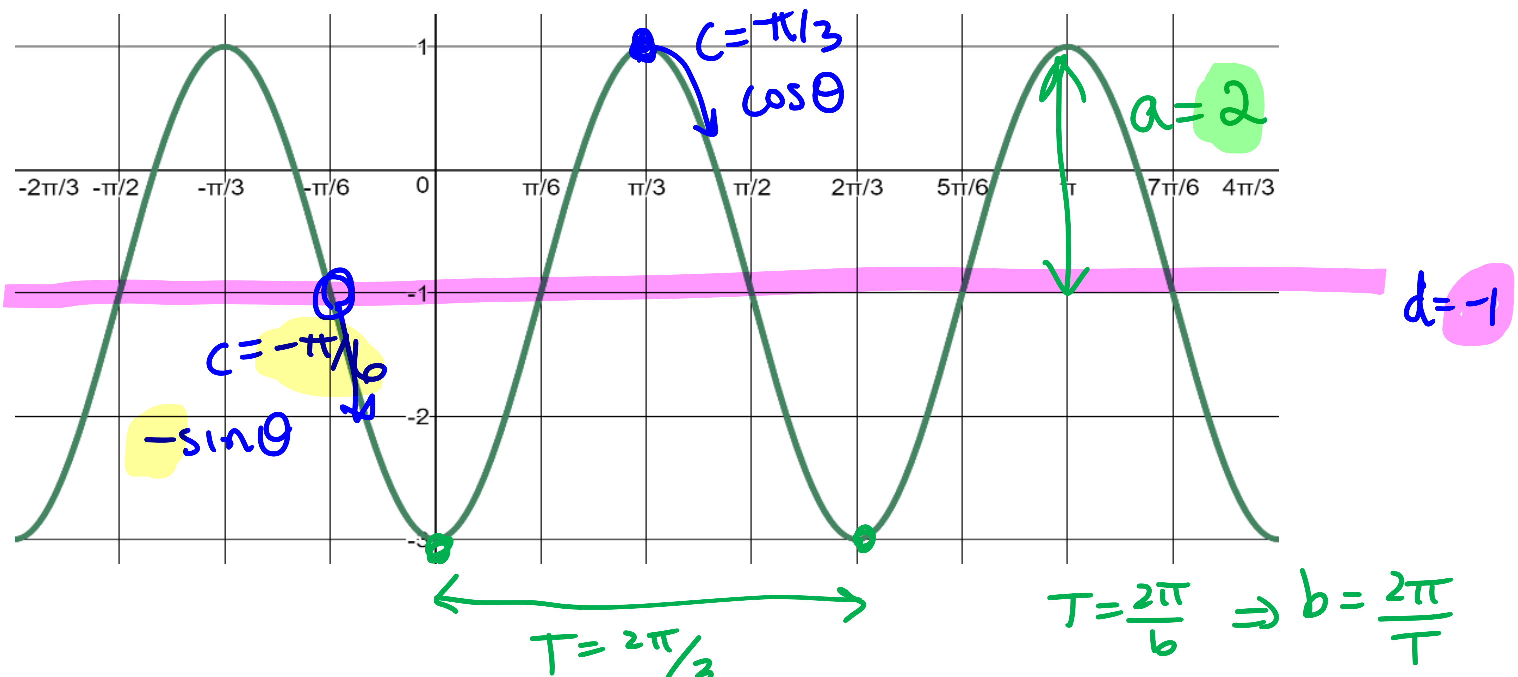
$T = \frac{2\pi}{b} = \frac{2\pi}{1/2}$

$= 4\pi$



midline

Example: Determine 3 different equations that could describe the following function.



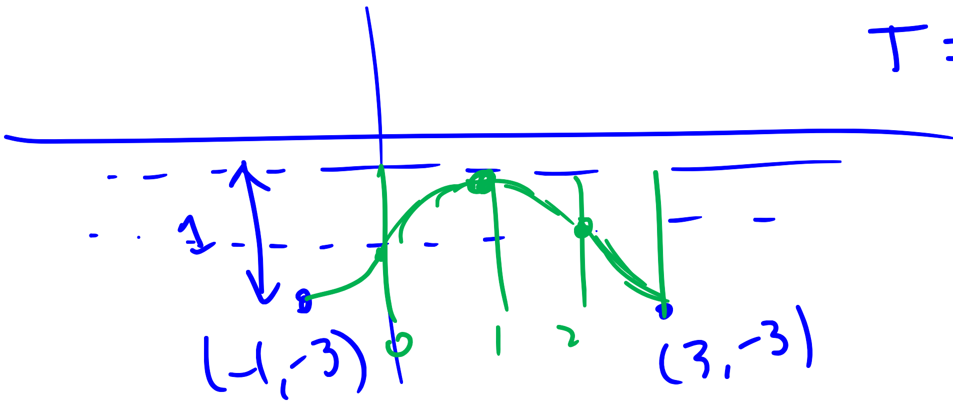
$T = \frac{2\pi}{b} \Rightarrow b = \frac{2\pi}{T}$

$b = \frac{2\pi}{2\pi/3} = 3$

$y_1 = -2\sin(3(x+\pi/6)) - 1$

$y_2 = 2\cos(3(x-\pi/3)) - 1$

Example: Determine two equations (one sine, one cosine) that could describe a sinusoidal function that has two minimums at $(-1,-3)$ and $(3,-3)$ and has an amplitude of 0.5.



$a = 0.5$

$T = 4 \Rightarrow b = \frac{2\pi}{4} = \frac{\pi}{2}$

$d = -2.5$

$$y_1 = 0.5\cos\left(\frac{\pi}{2}(x-1)\right) - 2.5$$

$$y_2 = -0.5\cos\left(\frac{\pi}{2}(x+1)\right) - 2.5$$

$$y_3 = 0.5\sin\left(\frac{\pi}{2}(x-0)\right) - 2.5$$

$$y_4 = -0.5\sin\left(\frac{\pi}{2}(x-2)\right) - 2.5$$

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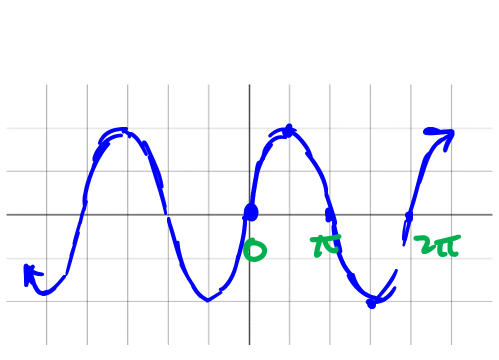
Trig Graphs

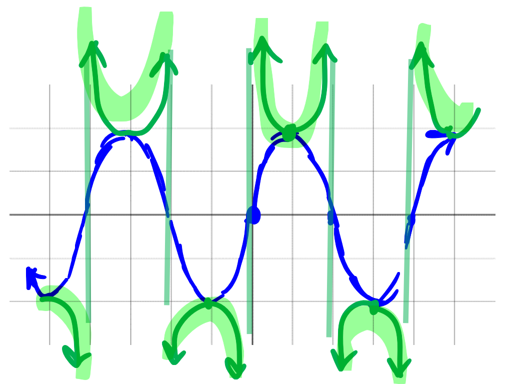
$\sin x$



Domain: $x \in \mathbb{R}$

Range: $y \in [-1, 1]$

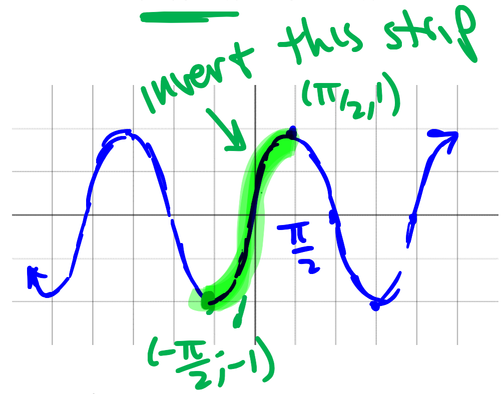
$\csc x$



Domain: $x \in \mathbb{R} \setminus \{n\pi\}$ $n \in \mathbb{Z}$

Range: $y \in (-\infty, -1] \cup [1, \infty)$

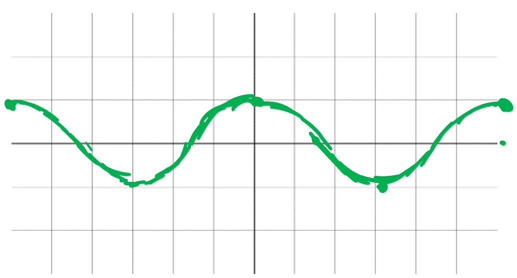
$\arcsin x = \sin^{-1} x$



Domain: $x \in [-1, 1]$

Range: $y \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

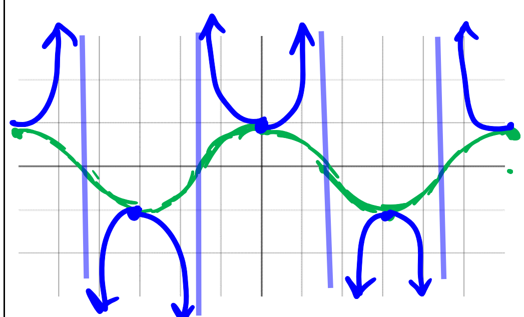
$\cos x$



Domain: $x \in \mathbb{R}$

Range: $y \in [-1, 1]$

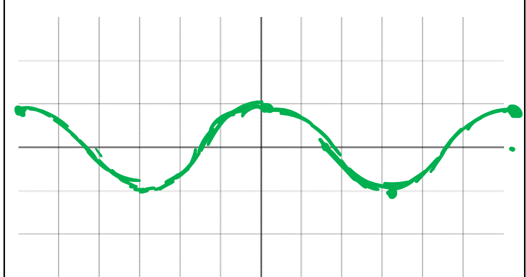
$\sec x$



Domain: $x \in \mathbb{R} \setminus \{\frac{2n+1}{2}\pi\}$

Range: $y \in (-\infty, 1] \cup [1, \infty)$

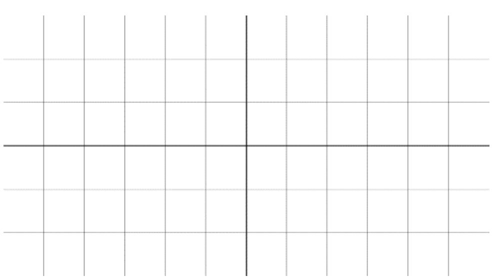
$\arccos x = \cos^{-1} x$



Domain:

Range:

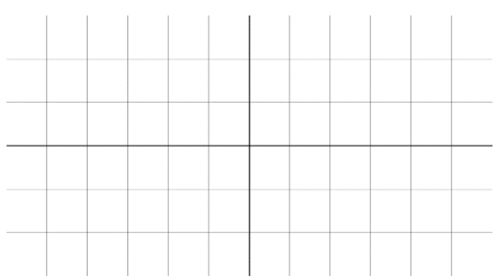
$\tan x$



Domain:

Range:

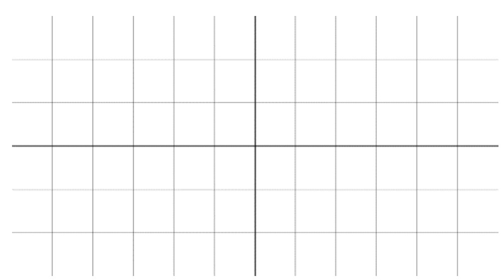
$\cot x$



Domain:

Range:

$\arctan x = \tan^{-1} x$



Domain:

Range: