Trig Modelling Practice

The following problems are snippets from the textbook with the core function erased. Determine the general function for them, use the function to predict a value, and use the function to solve a problem.

1.

a. A point on saw blade experiences motion around a circle with radius r. It makes n rotations per second. The blade sits so that the center is d units below the top of the table. Determine a function for the height of the point at time t. State the mapping notation of your function and describe its domain.

$$h(r, n, d, t) = r\cos(2\pi n \cdot t) - d$$

$$h: \mathbb{Q}_+ \times \mathbb{N} \times \mathbb{Q}^2 \to \mathbb{Q} \cap [-r-d, r-d]$$

$r > 0$, $d \geq 0$

b. If the radius is 10 cm and it sits 8 cm below the top of the table, what percentage of one rotation will the point be above the table?

$$h(10, n, 8, t) = 10\cos(2\pi n t) - 8 = 0$$

$$\Rightarrow \cos\theta = 8/10 \Rightarrow \theta = \pm 0.644 + 2\pi m \quad m \in \mathbb{Z}$$

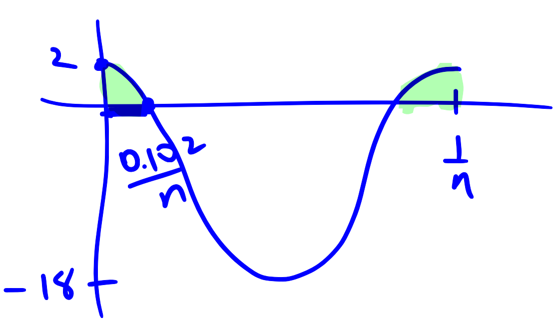
$$\Rightarrow \quad 2\pi n t = \pm 0.644 + 2\pi m$$

$$nt = \pm 0.102 + m$$

$$t = \pm \frac{0.102}{n} + \frac{m}{n}$$

time above is $\frac{0.204}{n}$ sec out of $\frac{1}{n}$

$\Rightarrow$ 20.4 % of the time we are above 0



2.

a. A satellite follows a sinusoidal path over the Earth in orbit. It takes the satellite m minutes to orbit the Earth. On one side of the Earth, it reaches a maximum height of h_1 (km) and on the opposite side is reaches a min height of h_0 (km). At t_0 minutes after noon, the satellite is at the min height. Determine a function for the height of the satellite at time t. State the mapping notation of your function and describe its domain.

$$h(h_1, h_0, m, t_0, t) = -\frac{h_1 - h_0}{2} \cos\left(\frac{2\pi}{m}(t - t_0)\right) + \frac{h_1 + h_0}{2}$$

$$h: \mathbb{Q}_+^2 \times \mathbb{N} \times \mathbb{Q}^2 \longrightarrow [h_0, h_1] \cap \mathbb{Q}$$

$h_1, h_0 > 0$ since the satelite hasn't crashed.

$t_0 \in \mathbb{Q}$ since a satelite can travel multiple km per second so I want decimals. Same $t \in \mathbb{Q}$

b. If $m = 200$ minutes, $h_1 = 300$ km and $h_0 = 220$ km, and at 12: 47 pm the satellite is at the min height, determine the intervals of time from midnight to 6:00 am of that day that the satellite was more then 280 km above the Earth.

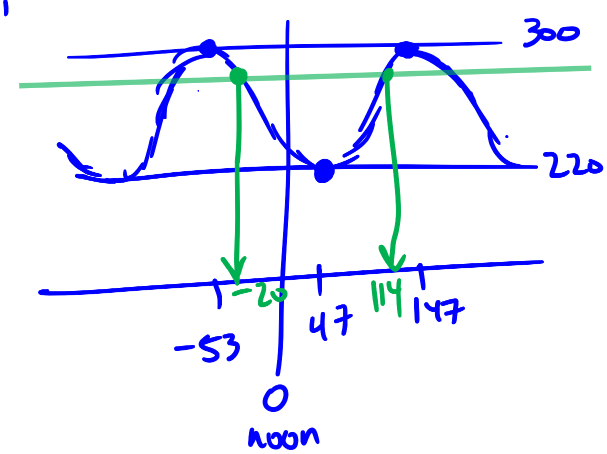
$$h(300, 220, 200, 47, t) = -40\cos\left(\frac{\pi}{100}(t - 47)\right) + 260 = 280$$

$$-40\cos\theta + 260 = 280$$

2, 1

$$\cos\theta = -\frac{1}{2} \Rightarrow \theta = \pm\frac{2\pi}{3} + 2\pi n \quad n \in \mathbb{Z}$$

$$\Rightarrow \frac{\pi}{100}(t - 47) = \pm\frac{2}{3}\pi + 2\pi n$$



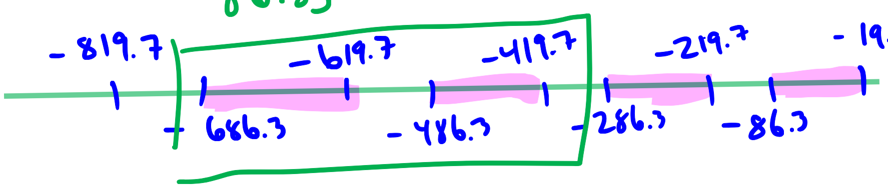
$$t - 47 = \pm\frac{200}{3} + 200n$$

$t = 113.67$ or $-19.67 + 200n$

-200

$= -86.33$

find $t \in [-720, -360]$



3h + 20 min = 1 period

The satelite was higher than 280km from:

12:33 am 41 sec – 1:40 am 19 sec

and

3:53am 41 sec – 5:00 am 19 sec

3.

a. The population of foxes in a region cycles from a minimum P_0 to maximum P_1 during a m month period (that is from P_0 to P_1 in m months). The population starts at P_1 on the first of month m_0. Determine a function for the population of foxes at time t in months. State the mapping notation of your function and describe its domain.

$$F(P_0, P_1, m, m_0, t) = \frac{P_0 - P_1}{2}\cos\left(\frac{\pi}{m}(t-m_0)\right) + \frac{P_0+P_1}{2}$$

$$F: \mathbb{N}^4 \times \mathbb{Q} \to \mathbb{N} \cap [P_0, P_1]$$

$m_0 \leq 12$ (since there are 12 months)

$t \in \mathbb{Q}$ since we could consider the population of foxes on March 15th (But $\mathbb{Z}$ would be a fine choice too)

b. If $P_0 = 600$ and $P_1 = 1600$, $m = 12$ months, and m_0 is March 2020, determine the approximate dates between Jan 1, 2020 to December 31, 2024, the population of foxes is greater than 1000.

$$F(600, 1600, 12, 3, t) = 500\cos\left(\frac{\pi}{12}(t-3)\right) + 1100 = 1000$$

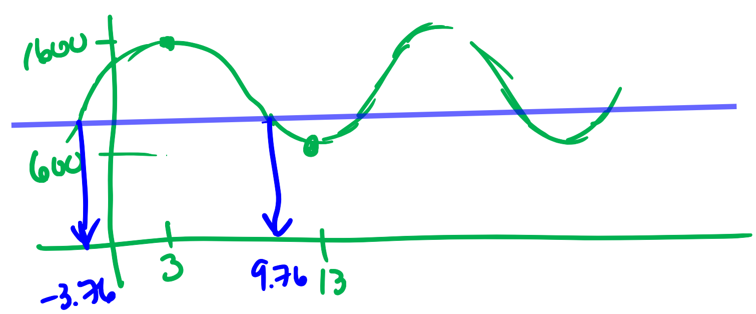
$5\cos\theta + 11 = 10$

$\theta = \pm 1.772 + 2\pi n \quad n \in \mathbb{Z}$

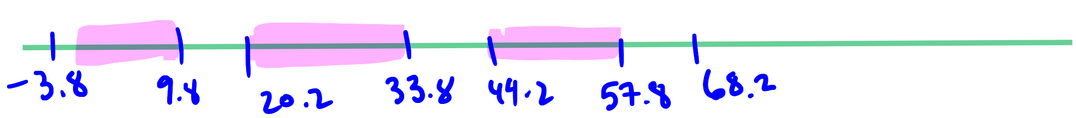
$\frac{\pi}{12}(t-3) = \pm 1.772 + 2\pi n$

$t - 3 = \pm 6.76 + 24n$

$t = 9.76$ or $-3.76 + 12n$



want $t \in [1, 13\times 5) = [1, 65)$



$\Rightarrow$ The fox population is above 1000 from

Jan 1 2020 – Sep 22 2020, Aug 24 2021 – Sep 22 2022

or Aug 24 2023 – Sep 22 2024

4.

a. The altitude of the Sun follows a sinusoidal path. The maximum altitude it reaches is θ_1 degrees above the horizon at time t_1 (hours). The lowest it reaches is θ_2 degrees below the horizon at time t_2 (hours). Determine a function for the height of the Sun as a function of time t. State the mapping notation of your function and describe its domain.

$$A(\theta_1, \theta_2, t_1, t_2, t) = \frac{\theta_1 - \theta_2}{2}\cos\left(\frac{\pi}{t_2 - t_1}(t - t_1)\right) + \frac{\theta_1 + \theta_2}{2}$$

$\theta_1 > \theta_2$

$$A: \mathbb{Z}^2 \times \mathbb{Q}^3 \to \mathbb{Z} \cap [\theta_1, \theta_2]$$

$\theta_1, \theta_2 \in [-90, 90]$, t, t_2 could be negative depending on the 0.

and $\theta_K \in \mathbb{Z}$

probably only to the nearest degree (making $\theta_K \in \mathbb{Q}$ would be fine too)

b. If $\theta_1 = 63°$ at 1:10 pm on June 3, 2021 and $\theta_2 = -18°$ at 1:10 am on June 4, 2021. Then Determine the time of sunrise and sunset on June 4, 2021.

$\underbrace{\text{sunrise and sunset}}_{\theta = 0}$

$t = 0$ is midnight June 4

$$A(63, -18, -10\tfrac{5}{6}, 1\tfrac{1}{6}, t) = 40.5\cos\left(\frac{\pi}{12}\left(t + 10\tfrac{5}{6}\right)\right) + 22.5 = 0$$

$\cos\theta = -5/9$

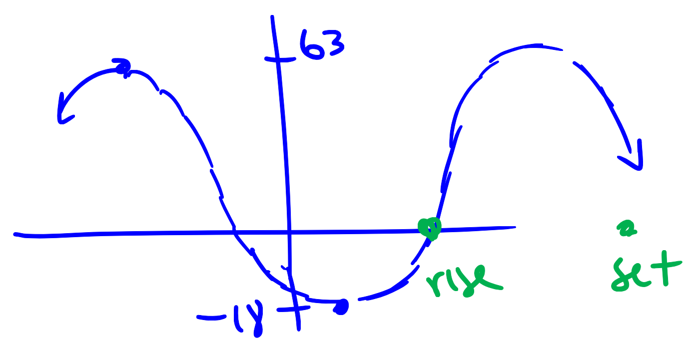
$\theta = \pm 2.16 + 2\pi n = \frac{\pi}{12}\left(t + 10\frac{5}{6}\right)$

$t + 10\frac{5}{6} = \pm 8.25 + 24n$

$t = -2.58$ or $-19.08 + 24n$

$+24$ $\qquad$ $+24$

$= 21.42$ $\qquad$ 4.92



Sun rise @ 4:55 am

Sun set @ 9:25 pm

5.

a. Daily temperature follows a sinusoidal curve. In Vancouver, it reaches a minimal temperature of T_0 degrees Celsius at time t_0 and a maximal temperature of T_1 at time t_1. Determine a function for the temperature as a function of the time t. State the mapping notation of your function and describe its domain.

$$T(T_0, T_1, t_0, t_1, t) = \frac{T_1 - T_0}{2}\cos\left(\frac{\pi}{t_1 - t_0}(t - t_1)\right) + \frac{T_1 + T_0}{2}$$

$$T: \mathbb{Z}^2 \times \mathbb{Q}^3 \to \mathbb{Z} \cap [T_0, T_1]$$

negative values should be fine for anything

$T_u \in \mathbb{Z}$ as the weather is usually to the nearest degree Celcius

b. If $T_0 = 15°\text{C}$ at 6:00 am and $T_1 = 28°\text{C}$ at 6:30 pm. Then determine the interval of times in the day when the temperature is above ~~25°C~~. 20°C

$$T(15, 28, 6, 18.5, t) = 6.5\cos\left(\frac{\pi}{12.5}(t - 18.5)\right) + 21.5 = 20$$

$\Rightarrow \cos\theta = -0.231$

$\theta = \pm 1.804 + 2\pi n \quad n \in \mathbb{Z}$

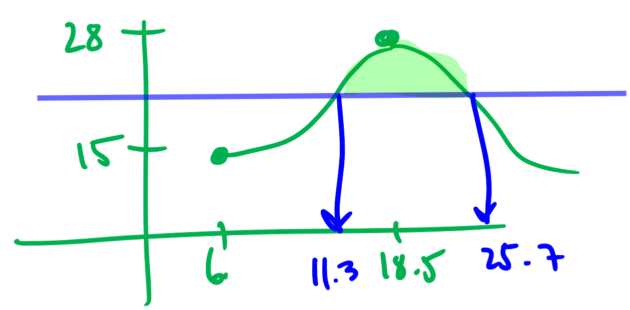
$\frac{\pi}{12.5}(t - 18.5) = \pm 1.804 + 2\pi n$

$t - 18.5 = \pm 7.177 + 25n$

$t = 25.68$ or $11.32 + 25n$

-25

0.68



In this day, the temperature is above 20°C from midnight – 12:40 am and 11:20 am – midnight

Generalize the scenarios in the textbook page 278-280 # 17-23. Think about a problem you could ask about them.