Function Transformations: Translations

Goals:
- Describe a horizontal and vertical translations in the form $T(x) = f(x - h) + k$.
- Understands that horizontal translations look to act in the opposite direction.
- Describe a horizontal and vertical stretch/compression/reflection in the form $T(x) = a \cdot f(bx)$
- Understands that the intercepts are invariant points after an expansion or reflection.

Terminology:
- Translation
- Mapping
- Image
- Expansion & Compression
- Reflection

$f(x) = \sqrt{x}$; $f(x) = x^3$; $f(x) = \sin(x)$

in out

Functions are operations. When we see the function $f(x) = y$ what is being denoted is a relationship of x to y and we can write it as follows using **mapping notation**

$$x \mapsto y \equiv (x, y)$$

For example: If $f(1) = 3$ and $f(2) = -1$ then

mapsto

$1 \mapsto 3$ $2 \mapsto -1$ $\equiv$

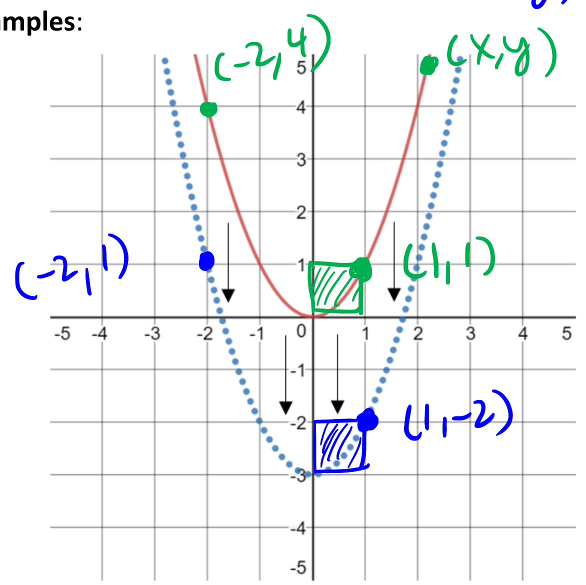
maps to

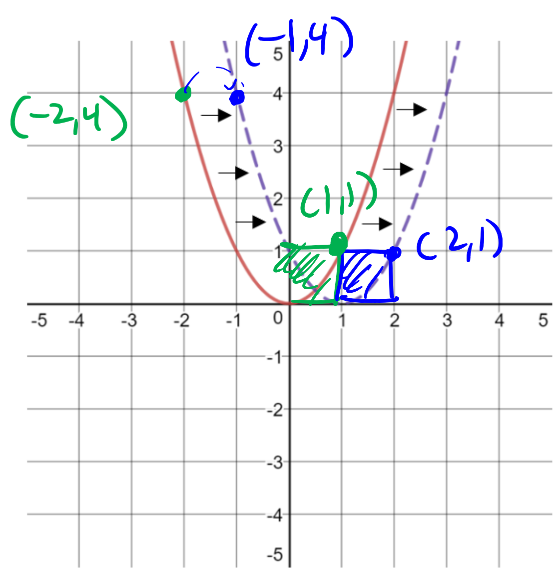
y (1,3) x (2,-1)

We are going to be looking at two major ways we can manipulate a function $f(x)$, and transform it into a new function, $T(x)$. For now, we will focus on just sliding the function around in 2D space (can move it horizontally and vertically). These are called **translations**.

$(x, y) \mapsto (u, v)$

Examples:





$(1,1) \mapsto (1,-2)$
$(-2,4) \mapsto (-2,1)$
$(x,y) \mapsto (x, y-3)$
shift down 3

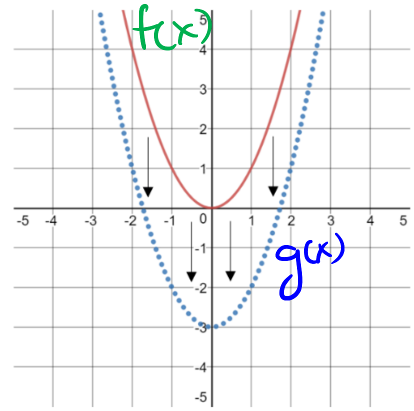
$(1,1) \mapsto (2,1)$
$(-2,4) \mapsto (-1,4)$
$(x,y) \mapsto (x+1, y)$
shift right 1

For a **vertical translation**, we take our original function where $y = f(x)$ and…

$(x, y) \longmapsto (x, y + d)$

vertical shift
$d > 0$ up
$d < 0$ down

$g(x) = f(x) + d$



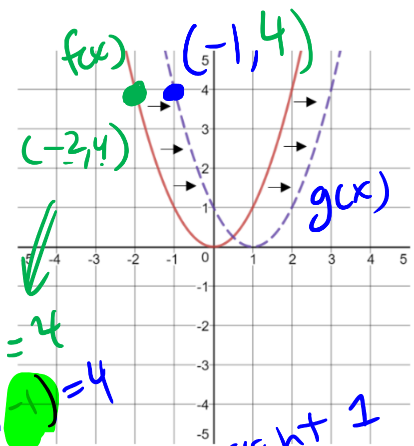
For a **horizontal translation**, we shift the function left and right but …

$(x, y) \longmapsto (x + c, y)$

change.

horiz. shift
$c > 0$ right
$c < 0$ left

$g(x) = f(x - c)$

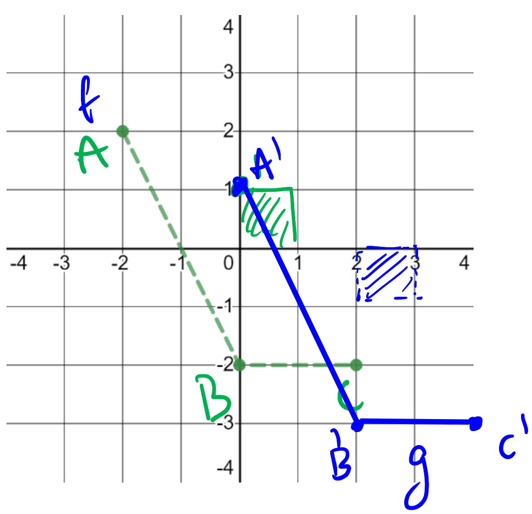


$f(-2) = 4$
$f(-1-1) = 4$

-1 means right 1

** After a transformation, the resulting function is called the **image function**.

Example 1: Given the graph of f, graph the image function after being translated 2 units right and 1 unit down. Write the mapping notation and function notation of the transformation



$(x, y) \longmapsto (x + 2, y - 1)$

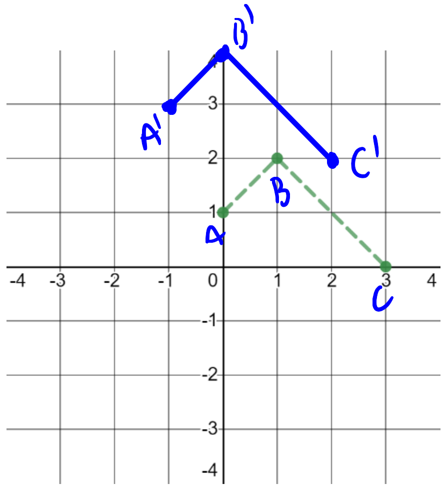
$(-2, 2) \longmapsto (0, 1)$

$(0, -2) \longmapsto (2, -3)$

$(2, -2) \longmapsto (4, -3)$

$g(x) = f(x - 2) - 1$

Practice: Given the graph of g, graph the image function after being translated 1 unit left and 2 units up. Write the mapping notation and function notation of the transformation



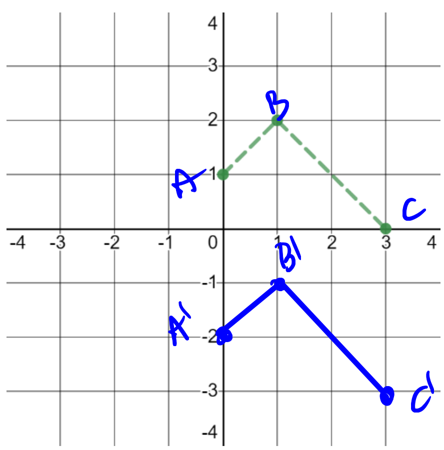
$(x, y) \mapsto (x-1, y+2)$

$T(x) = g(x+1) + 2$

$(0,1) \mapsto (-1,3)$

Practice: Given the graph of g, graph the image function after it has been translated as follows:

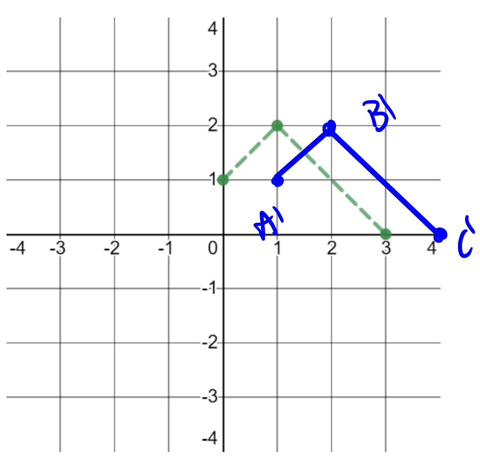
$$(x, y) \mapsto (x, y-3)$$



$T(x) = g(x) - 3$

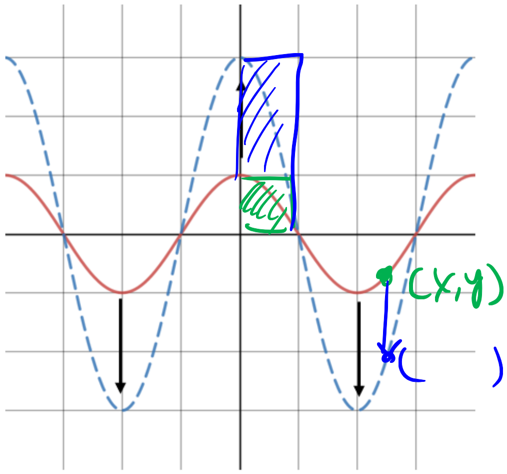
Practice: Given the graph of g, graph the image function after it has been translated as follows:

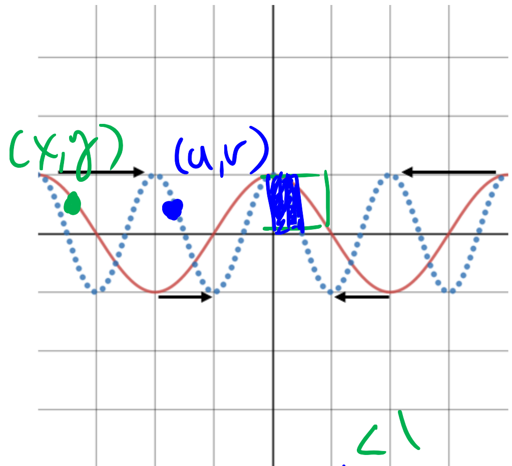
$$T(x) = g(x-1)$$



$(x, y) \mapsto (x+1, y)$

Aside from translating a function which preserves the general characteristics of the function (it just got moved around the graph) we can transform the graph in a more significant manner by stretching and compressing it relative to either axis.





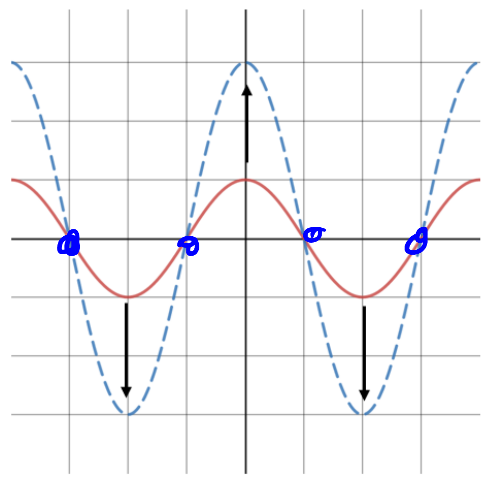
$(x,y) \mapsto (x, 3y)$ >1

expanded space vertically.

$(x,y) \mapsto (\frac{1}{2}x, y)$ <1

compressed space horizontally

For a **vertical expansion or compression** (expansion about the x-axis), we take our original function where $y = f(x)$ and...

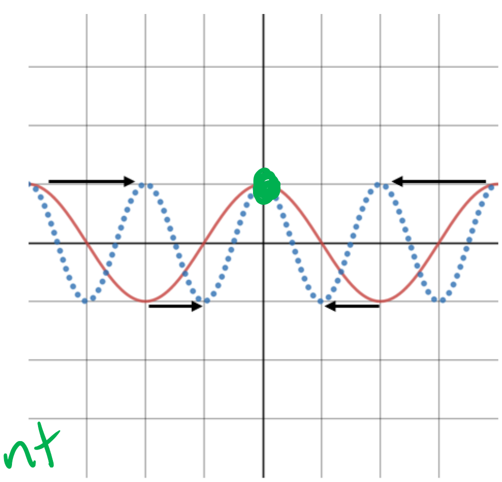


$$(x,y) \mapsto (x, ay)$$

$$g(x) = a f(x)$$

** $(x, 0) \mapsto$ $(x, a\cdot 0) = (x, 0)$

For a **horizontal expansion or compression** (expansion about the y-axis), we have that...



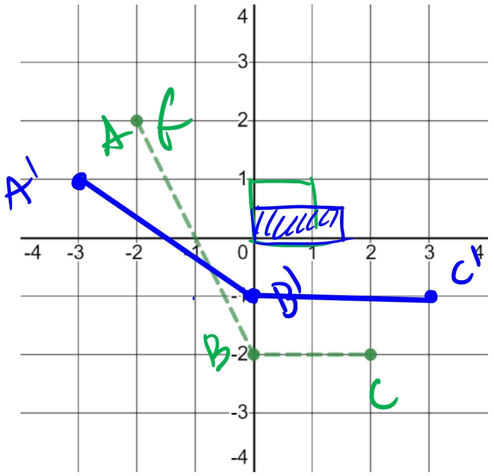
$$(x,y) \mapsto (bx, y)$$

$$g(x) = f(\frac{1}{b}x)$$

opposite when inside f.

** $(0, y) \mapsto$ $(b0, y) = (0, y)$ invarient point

Example 2: Given the graph of f, graph the image function after it has vertically been compressed by a factor of 2 and horizontally expanded by a factor of $\frac{3}{2}$. Write the mapping notation and function notation of the transformation.



$(x,y) \mapsto \left(\frac{3}{2}x, \frac{1}{2}y\right)$

$(-2,2) \mapsto (-3,1)$

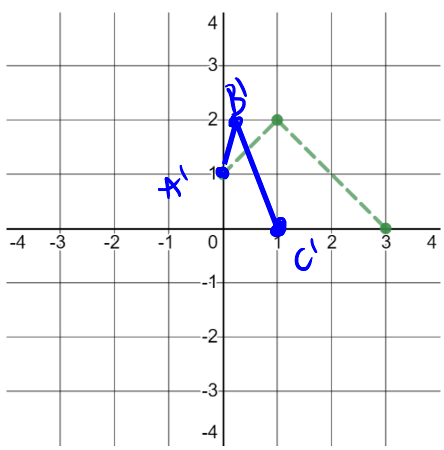
$(0,-2) \mapsto (0,-1)$

$(2,-2) \mapsto (3,-1)$

$g(x) = \frac{1}{2}f\left(\frac{2}{3}x\right)$

Practice: Given the graph of g, graph the image function after it has been translated as follows:

$$(x,y) \mapsto \left(\frac{1}{3}x, y\right)$$



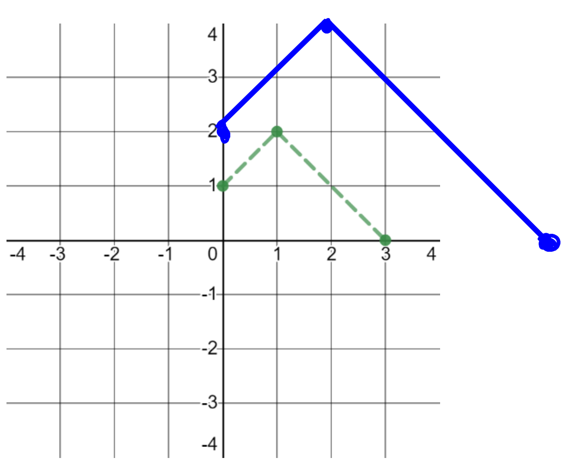
$T(x) = g(3x)$

$(0,1) \mapsto (0,1)$ $(3,0) \mapsto (1,0)$

$(1,2) \mapsto (\frac{1}{3}, 2)$

Practice: Given the graph of g, graph the image function after it has been translated as follows:

$$T(x) = 2g\left(\frac{1}{2}x\right)$$



$(x,y) \mapsto (2x, 2y)$

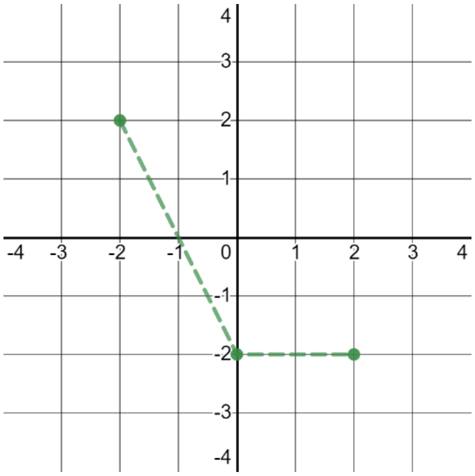
$(0,1) \mapsto (0,2)$

$(1,2) \mapsto (2,4)$

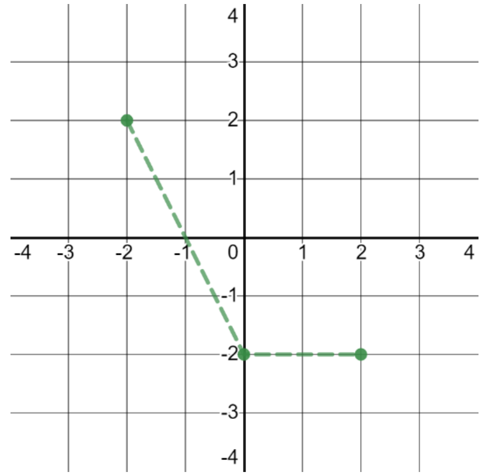
$(3,0) \mapsto (6,0)$

If the value of a or b is negative, this means we have the cases of a **reflection**.

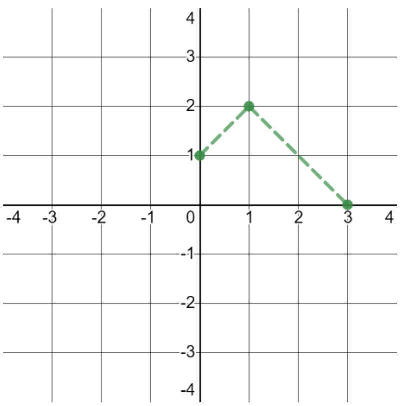
$$(x, y) \mapsto (x, -y)$$



$$(x, y) \mapsto (-x, y)$$

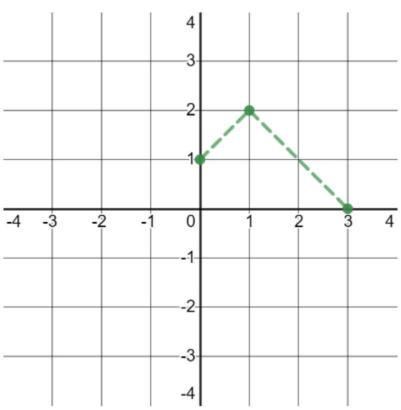


Practice: Given the graph of g, graph the image function after it has been reflected over the y-axis. Write the mapping notation and function notation of the transformation.



Practice: Given the graph of g, graph the image function after it has been translated as follows:

$$T(x) = -\frac{3}{2}g(-x)$$



Suggested problems: 1.1 page 12 – 14 # 2-4, 8-12, 16, 18, 19, C1 1.2 page 28 – 31 # 3-5, 7, 10, 12, 14, 16, C1, C2, C3
Textbook Reading: 1.1 page 6-12 & 1.2 page 16-27 **Key Ideas** on page 12 and 27
Next Class: Combining transformations and identifying transformed graphs