

Exponential Functions

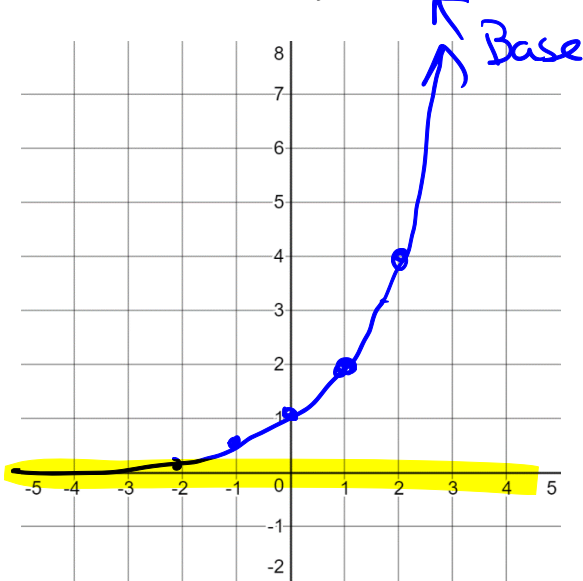
Goal:

- Can graph exponential functions of the form $f(x) = base^{x/T}$
- Understands that *base* is the rate of growth/decay and *T* is the length of one growth/decay period
- Can transform the exponential function in the form $f(x) = a(base)^{b(x-c)} + d$ to fit the need of the problem
- Can sketch e^x

Terminology:

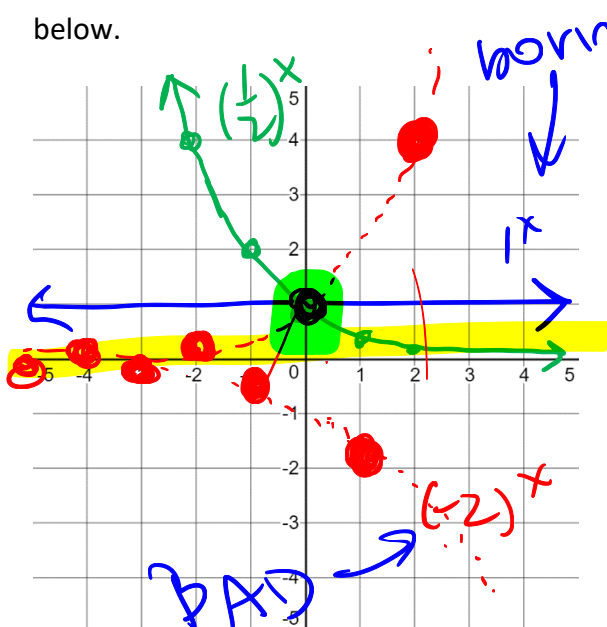
- Exponential growth/decay
- Euler's number

Consider the equation $f(x) = 2^x$. Graph it using a table of values and describe the characteristics.



x	y
-2	$2^{-2} = (\frac{1}{2})^2 = \frac{1}{4}$
-1	$2^{-1} = \frac{1}{2}$
0	$2^0 = 1$
1	2
2	4

Do the same for the functions $g(x) = 1^x$; $h(x) = (\frac{1}{2})^x$; and $k(x) = (-2)^x$ and graph it on the same graph below.



x	1^x
-2	1
-1	1
0	1
1	1
2	1

x	$(\frac{1}{2})^x$
-2	4
-1	2
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$

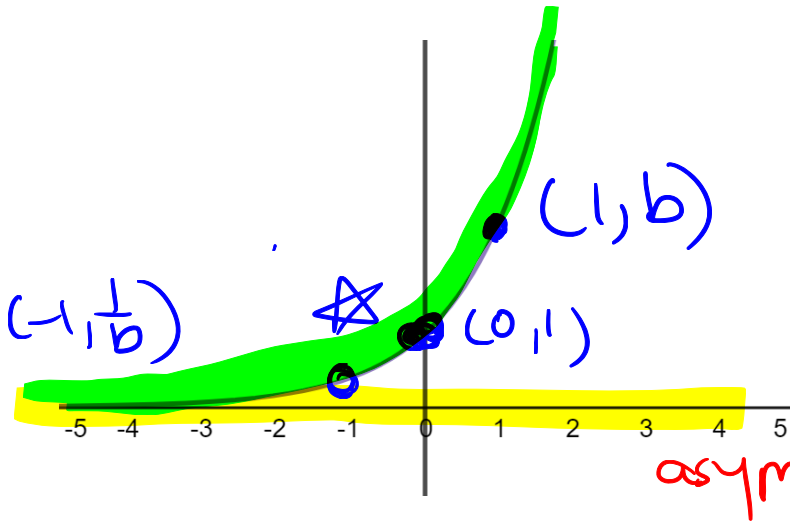
x	$(-2)^x$
-2	$\frac{1}{4}$
-1	$-\frac{1}{2}$
0	1
1	-2
2	4

$(-2)^{0.5} = (-2)^{1/2} = \sqrt{-2}$
 $(-2)^{1/3} = -1.26$

Exponential Growth

Graphing Exponential Functions

The general graph form $y = b^x$ for $b > 1$



$f(x) = b^x$

RoY $b > 1$

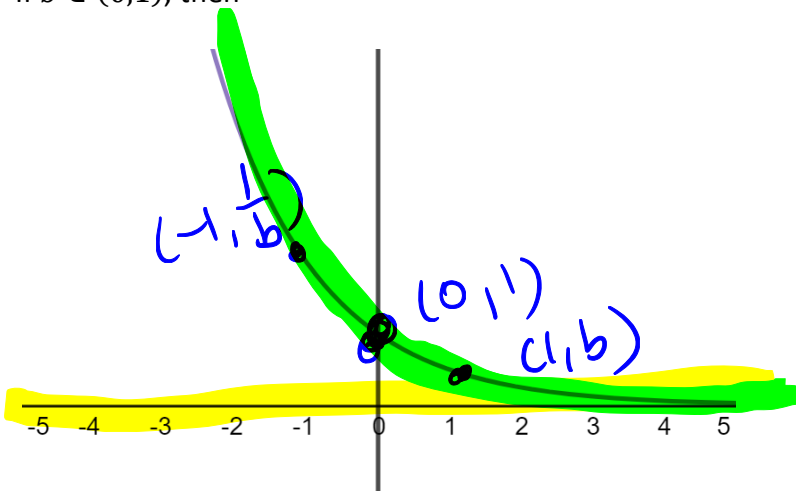
$g(x) = f(-x)$

$(x, y) \mapsto (-x, y)$

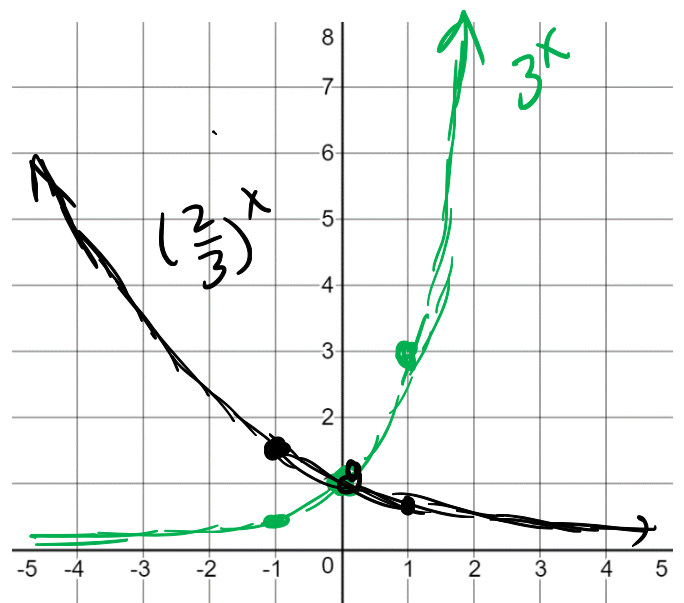
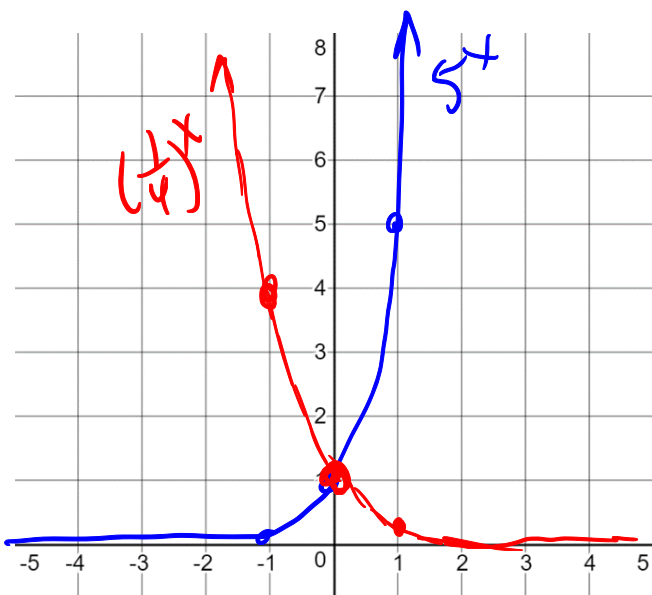
$f(-x) = b^{-x} = (\frac{1}{b})^x$

$\frac{1}{b} < 1$

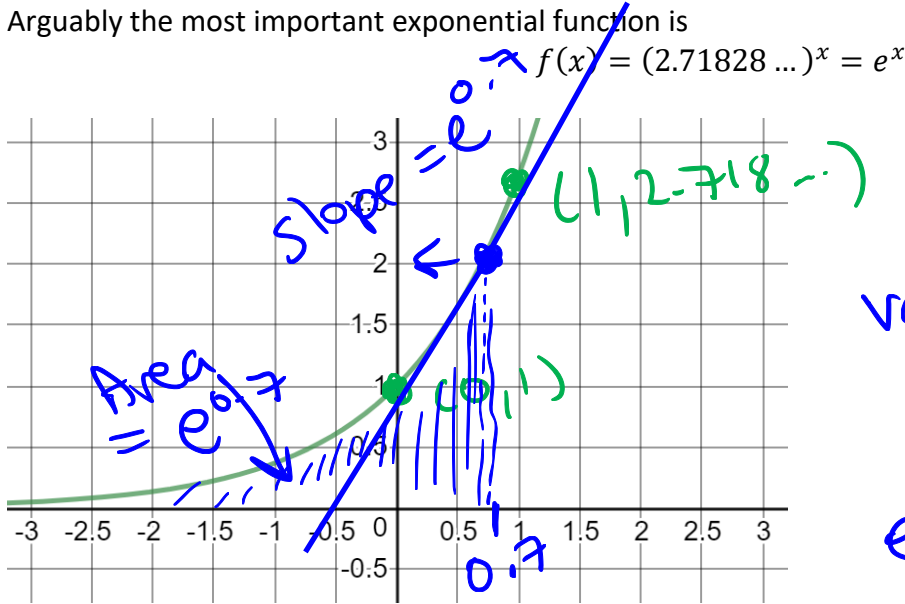
If $b \in (0, 1)$, then



Practice: Graph the functions $f(x) = 5^x$; $g(x) = (\frac{1}{4})^x$; $h(x) = 3^x$; $k(x) = (\frac{2}{3})^x$



Arguably the most important exponential function is



Euler's number
0.7er

value = slope = Area

e^x is THE exponential

$e^{0.7} = 2.01 \approx 2$

The beauty is that we can change from one base to another.

e^x e $f(x) = 2^x = (e^{0.7})^x = e^{0.7x}$

~~E~~

We'll talk more about this Monday on finding that power for e , but right now just be comfortable that we can do this at least through guess and check and when you start doing calculus you will only want to work with e .

The application of exponential functions is when we have populations of something that grow proportionally to the current size of the population (ie, bacteria growth, radioactive decay, information spreading)

Consider the specific case where a population doubles every year.

Population	1	2	4 = 2 ²	8 = 2 ³	16 = 2 ⁴	...	2 ^t
Time	0	1	2	3	4	...	t

$(time, pop) = (0, 1)$

And then the general case where a population grows at a rate r and it takes T years to grow that much or that little.

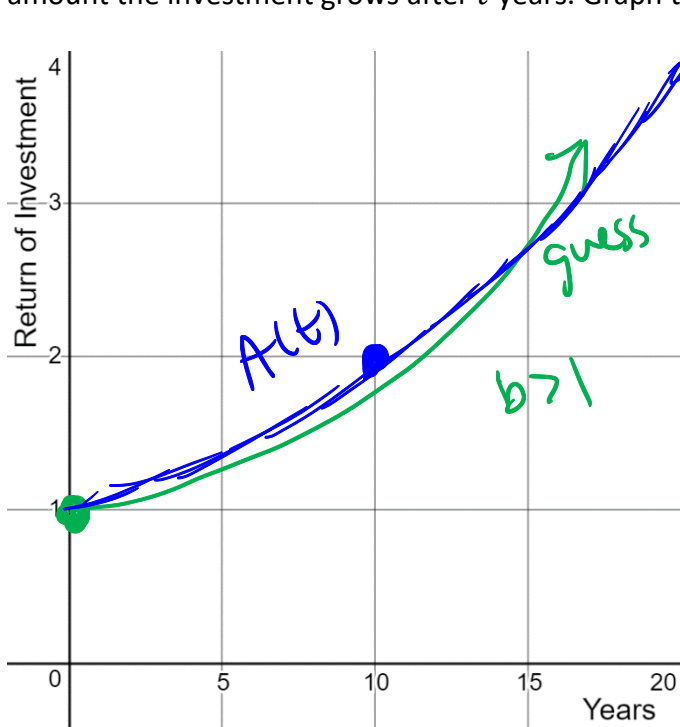
Population	1	r	r^2	r^3	r^4	...	r^n
Time	0	T	$2T$	$3T$	$4T$...	$nT = t$

$r^n = r^{t/T} = f(t)$

Basic exponential

$n = \frac{t}{T}$

Example: A given investment is expected to return 7% annually (every year). Determine an equation for the amount the investment grows after t years. Graph the equation for the given domain.



$r = ? = 1 + 0.07 = 1.07$
 $T = ? = 1 \text{ year}$

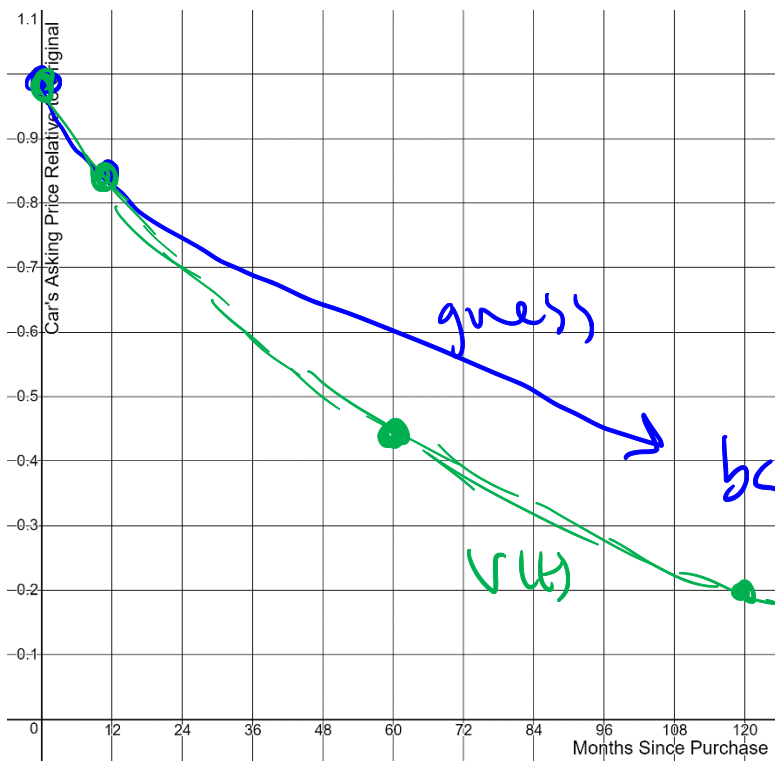
$$A(t) = (1.07)^{\frac{t}{1}}$$

$$= (1.07)^t$$

$$A(10) = 2$$

$$A(20) = 3.9$$

Practice: The price of a car decreases in price by about 15% every 12 months. Determine an equation for the car's value compared to the buying price after m months. Graph the equation for the given domain



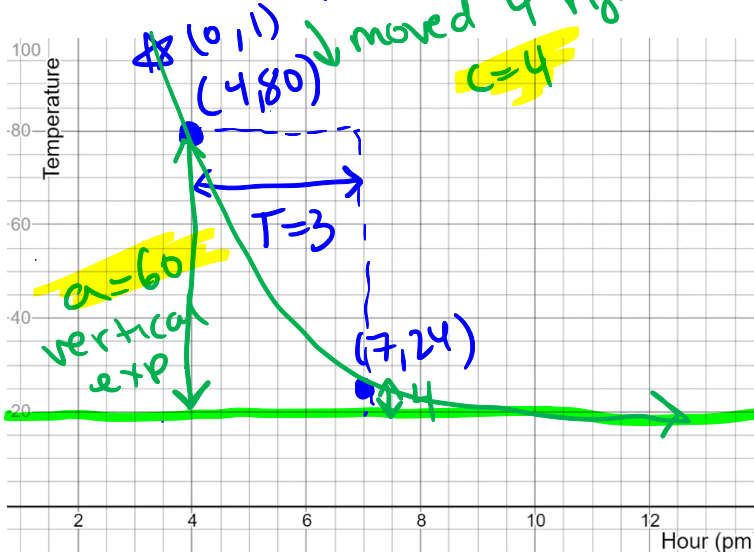
$T = 12 \text{ months}$
 $r = 1 - 0.15 = 0.85$

$$V(t) = (0.85)^{t/12}$$

$$V(60) = 0.44$$

$$V(120) = 0.20$$

Example: A baked pie comes out of the oven and is 80°C at 4pm, it is left to cool until dinner at 7pm when the temperature is 24°C . If the temperature of the room is 20°C then make an equation for the temperature of the pie at time t (pm)



$$60 \xrightarrow{+} 4 \equiv 60^r = 4$$

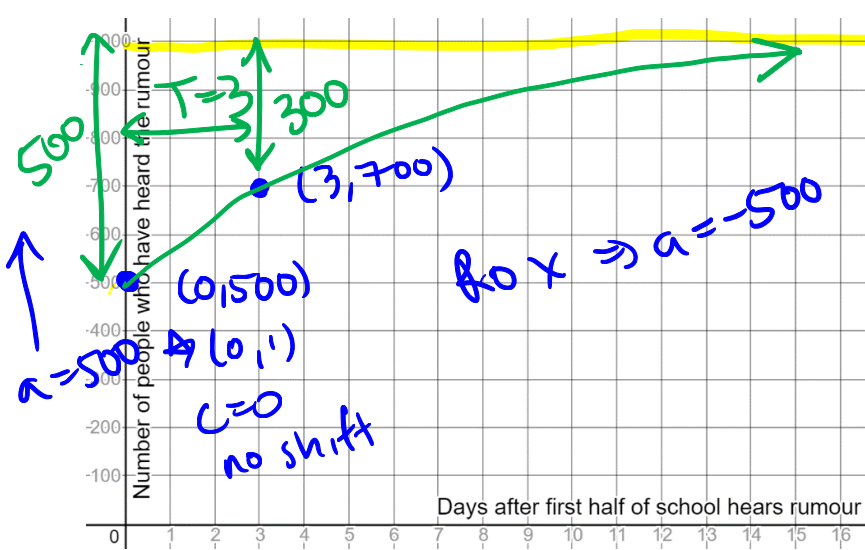
$$r = \frac{4}{60} = \frac{1}{15}$$

Basic: $\left[\left(\frac{1}{15} \right)^{t/3} \right] = f(t)$

$$P(t) = a f(t-c) + d$$

$$= 60 \left(\frac{1}{15} \right)^{\frac{t-4}{3}} + 20$$

Practice: After a rumour has initially spread, the number of new people who hear the rumour decreases exponentially. Prince of Wales has about 1000 students and after 500 people have heard the rumour it takes 3 days for another 200 to hear it. Determine an equation that describes the number of people who have heard the rumour at time t after the first 500 people hear it.



$$y = 1000 = d$$

$$500 \xrightarrow{+} 300$$

$$500^r = 300$$

$$r = \frac{3}{5}$$

Basic: $f(t) = \left(\frac{3}{5} \right)^{t/3}$

$$R(t) = a f(t-c) + d$$

$$= -500 \left(\frac{3}{5} \right)^{\frac{t}{3}} + 1000$$

<p>Suggested Practice Problems: 7.1 page 342-345 # 4-10, 14 7.2 page 354-357 # 3, 7-12</p>
<p>Textbook Reading: 7.1 and 7.2 page 334-342 and 346-354 Key Ideas on page 342 and 354</p>
<p>Next Class: Log function</p>