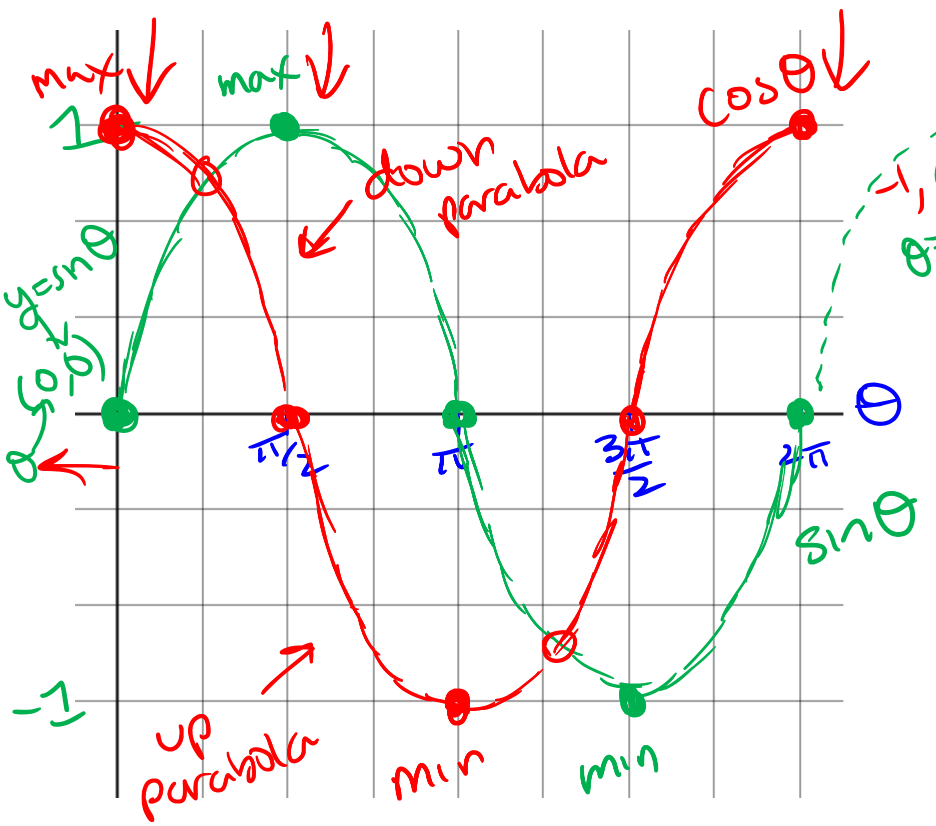
Graphing Sine, Cosine, and Tangent

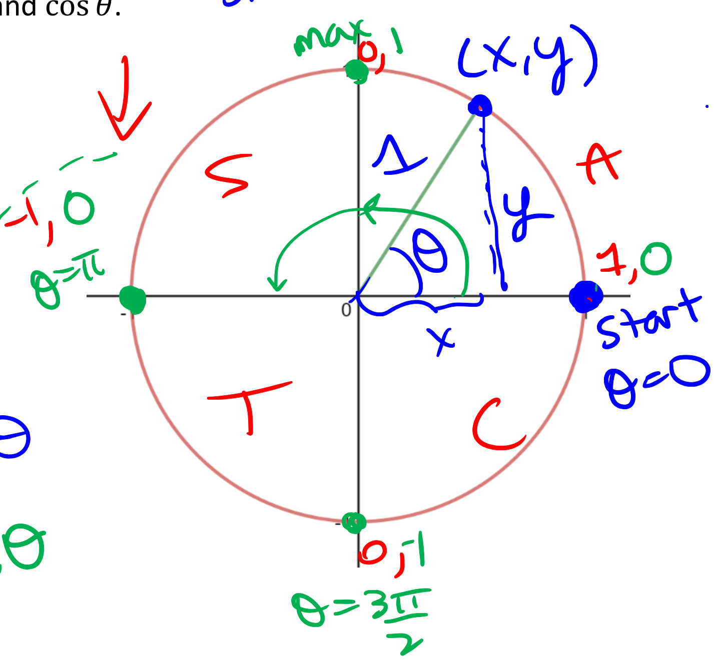
Goal:
- Can quickly graph/recognize the graph of cosine and sine and manipulate their amplitudes and periods
- Knows the general form of $a\sin(bx)$ and key points along the curve.
- Can graph $\tan\theta$ and describe its basic characteristics (period, zeros, asymptotes)

Terminology:
- Amplitude
- Period
- Sinusoidal curve
- Periodic function

Show $\sin\theta = y$, $\cos\theta = x$

Using a unit circle, graph the angle θ and the values of $\sin\theta$ and $\cos\theta$.





$\sin\theta = \frac{y}{1} = y$

$\cos\theta = \frac{x}{1} = x$

Characteristics:

→ If θ goes clockwise we go -ve direction (θ-axis)

→ $\cos\theta$ is symmetric over y-axis $\cos(-\theta) = \cos\theta$

→ $\frac{\pi}{4} + \pi n$ $\sin\theta$ and $\cos\theta$ intersect.

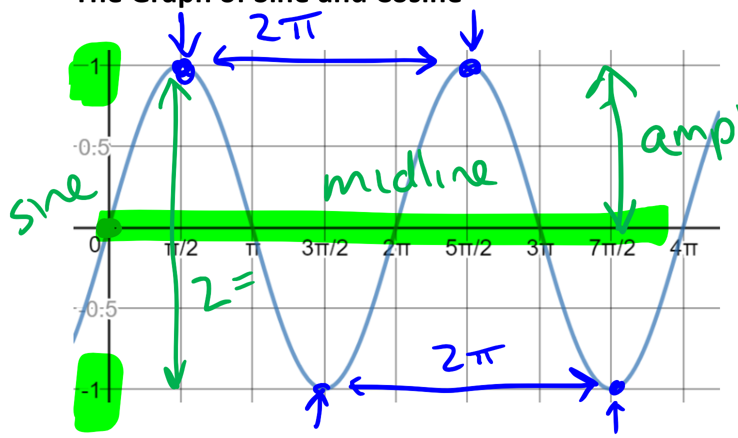
→ Same wavelength (?) ⇒ same gap between peeks. $\sin\theta$ & $\cos\theta$ shifted

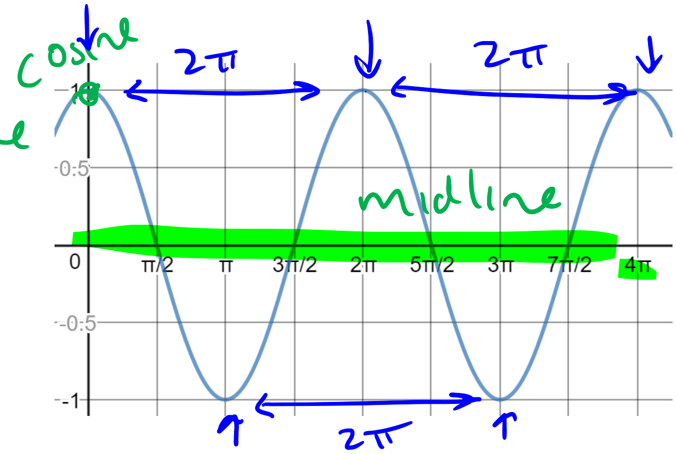
→

Definition: Functions that repeat after a certain amount of time are called **periodic functions** (periodic meaning occurring at regular intervals). Periodic functions that have this "wave" shape are called **sinusoidal functions**.

Period $= 2\pi = T$

The Graph of Sine and Cosine





We want to analyze this curve so that we can graph functions of the form

change the radius of the unit circle ← $a \cdot \sin b\theta$ and $a \cdot \cos b\theta$

change period

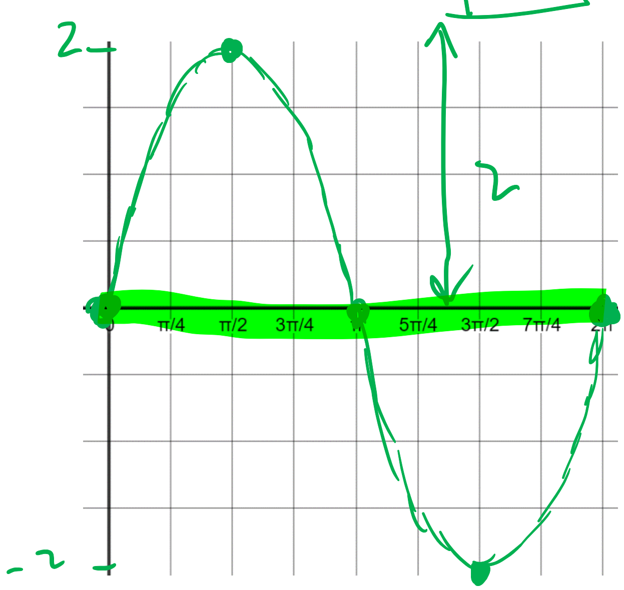
$T = \frac{2\pi}{b}$

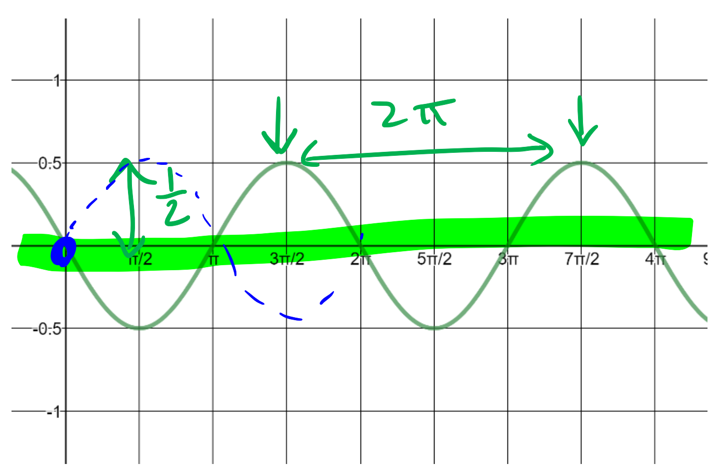
Definition: The *amplitude* is the distance from the midline to the maximum or minimum, or equivalently, half the distance between the max and min.

For sine and cosine, The normal amplitude is 1 and $T = 2\pi$

for $a \sin b\theta$, amplitude is a and $T = \frac{2\pi}{b}$

Example: Graph the function $\boxed{2 \sin \theta}$ and determine the equation to the other graph.





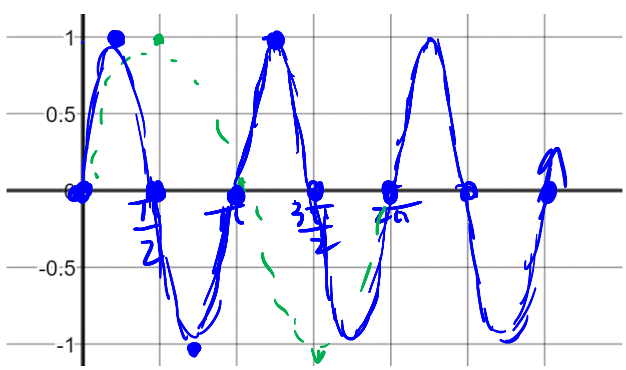
$-\frac{1}{2}\sin\theta$ or $\frac{1}{2}\sin(-\theta)$

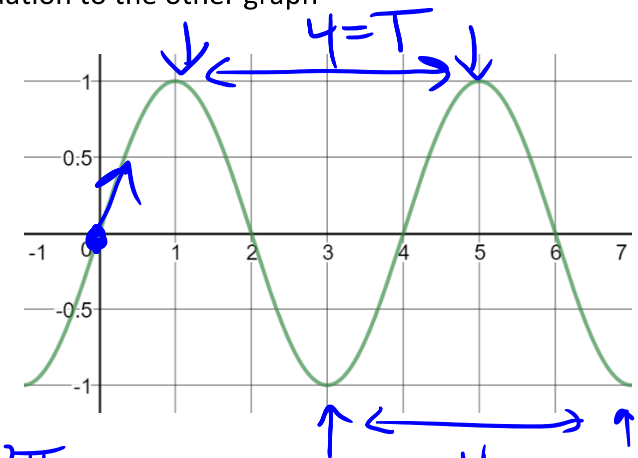
Definition: The *period* is the length of one complete cycle of a periodic function. Not necessarily how long it takes to repeat itself, but how long it takes to repeat the pattern.

For sine and cosine,

$$T = \frac{2\pi}{b}$$

Example: Graph the equation $\sin(2\theta)$ and determine the equation to the other graph



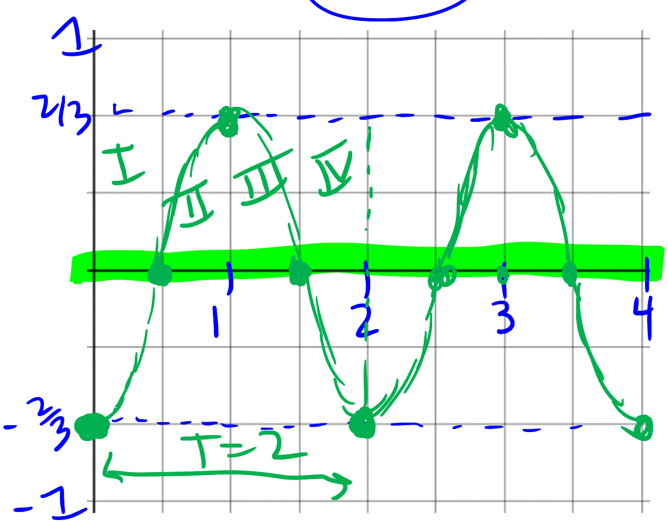


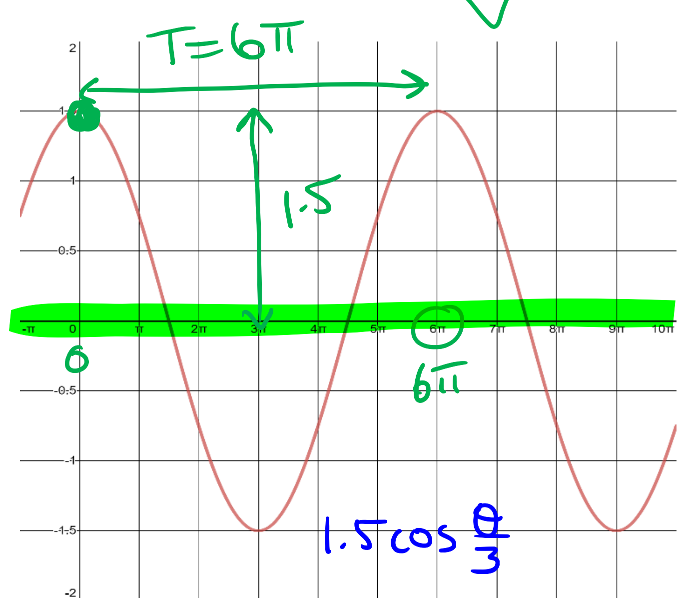
$T = \pi$

$4 = \frac{2\pi}{b} \Rightarrow b = \frac{\pi}{2}$

$\sin\left(\frac{\pi}{2}\theta\right)$

Practice: Graph $-\frac{2}{3}\cos(\pi\theta)$ and determine the equation to the other graph.





amplitude = $+\frac{2}{3} > 0$

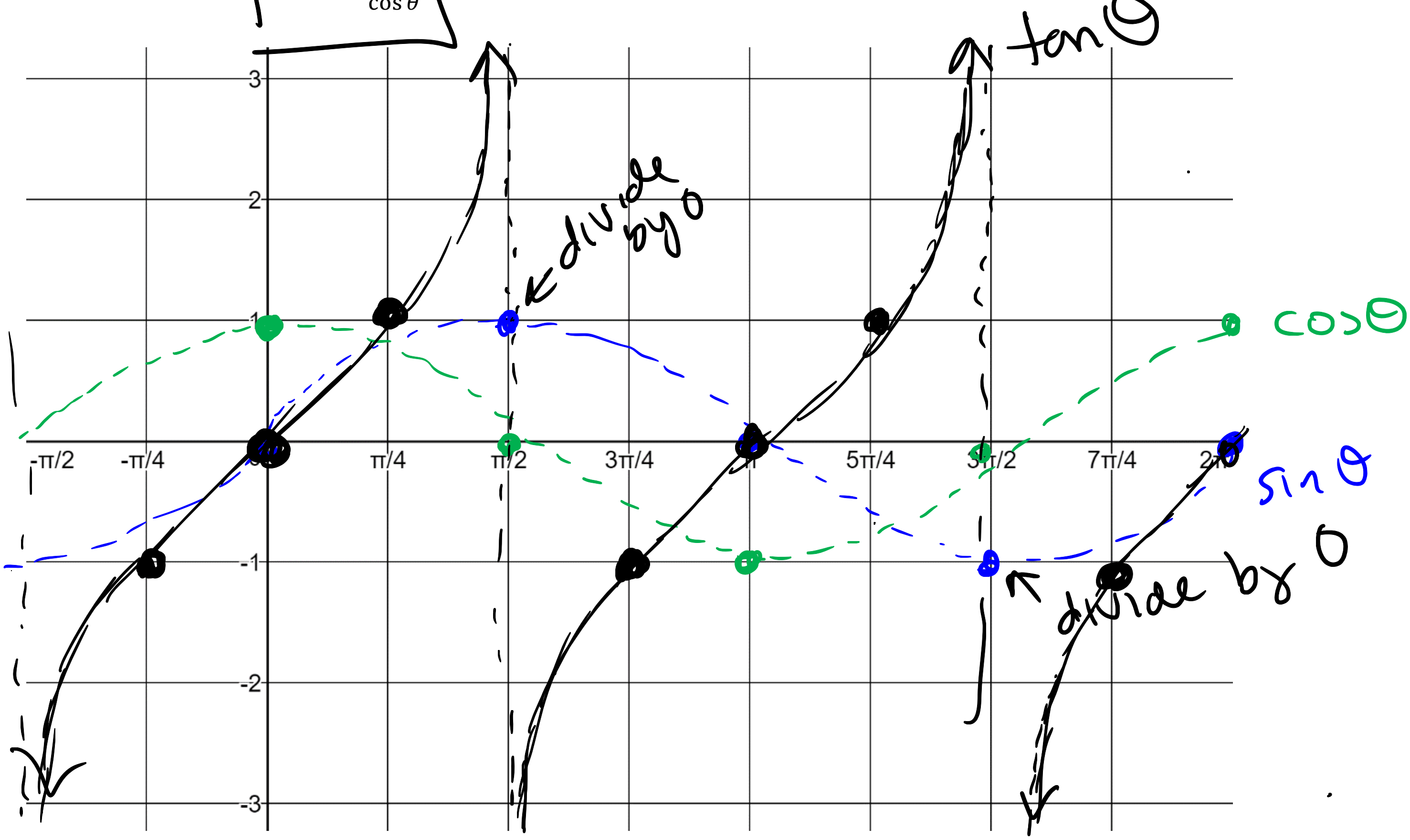
$T = \frac{2\pi}{\pi} = 2$

$1.5\cos\frac{\theta}{3}$

$a = 1.5$ $b = \frac{1}{3}$

$T = \frac{2\pi}{b} = 6\pi$

Using the fact that $\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$ graph tangent below:



Characteristics:

→ Period is π

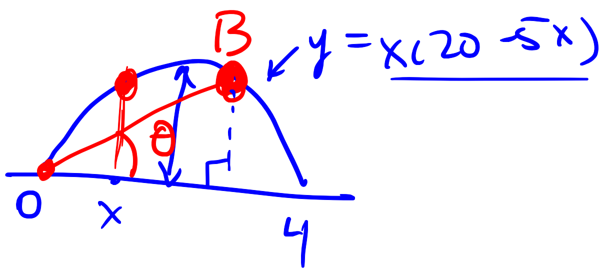
→ zeros @ $\theta = \pi n, n \in \mathbb{Z}$ (same as $\sin\theta$)

→ asymptotes @ $\theta = \frac{\pi}{2} + \pi n, n \in \mathbb{Z}$ (zeros of $\cos\theta$)

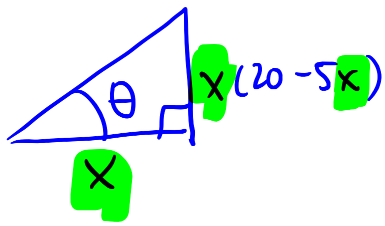
→ a ROY is the same as ROX

The textbook uses this section to introduce you into building trig equations that model moving lengths of triangles (an important part of calculus called **related rates**).

Example: A baseball is hit and travels along the curve $y = x(20 - 5x)$ where y is the vertical distance and x is the horizontal distance from the ball to the batter. Relate the angle the baseball makes with the batter and the horizontal distance away from the batter.



$$\tan\theta = \frac{x(20-5x)}{x}$$

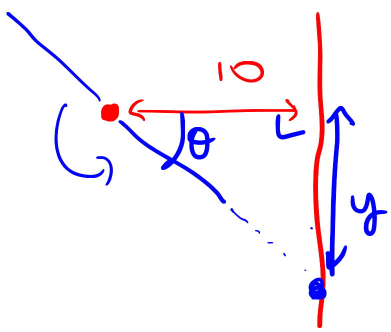


$$\Rightarrow \tan\theta = 20 - 5x$$

$$\theta = \tan^{-1}(20-5x) = \arctan(20-5x)$$

Practice: A police light shines light in two opposite directions and makes a full revolution every 1 second. If the light is positioned 10m from a wall write an expression for the distance the light travels along the wall as a function of the angle and then as a function of time.

change in θ is 2π rad/sec



$$\tan\theta = \frac{y}{10}$$

$$y = 10\tan\theta = 10\tan(2\pi t)$$

$$\theta(t) = 2\pi t + \pi$$

$$\theta(t) = 2\pi t$$

$\theta(0) = 0$ $\theta(1) = 0 = 2\pi$ $\theta(0.5) = \pi$

Suggested Practice Problems: 5.1 # 4-10, 13-15, 20 5.3 # 3, 5, 8-12
Textbook Reading: 5.1 and 5.3 page 222-231 and 256-261. Key Ideas page 232 and 262
Next Class: Transformations of Trig Function