Graphing Sine, Cosine, and Tangent

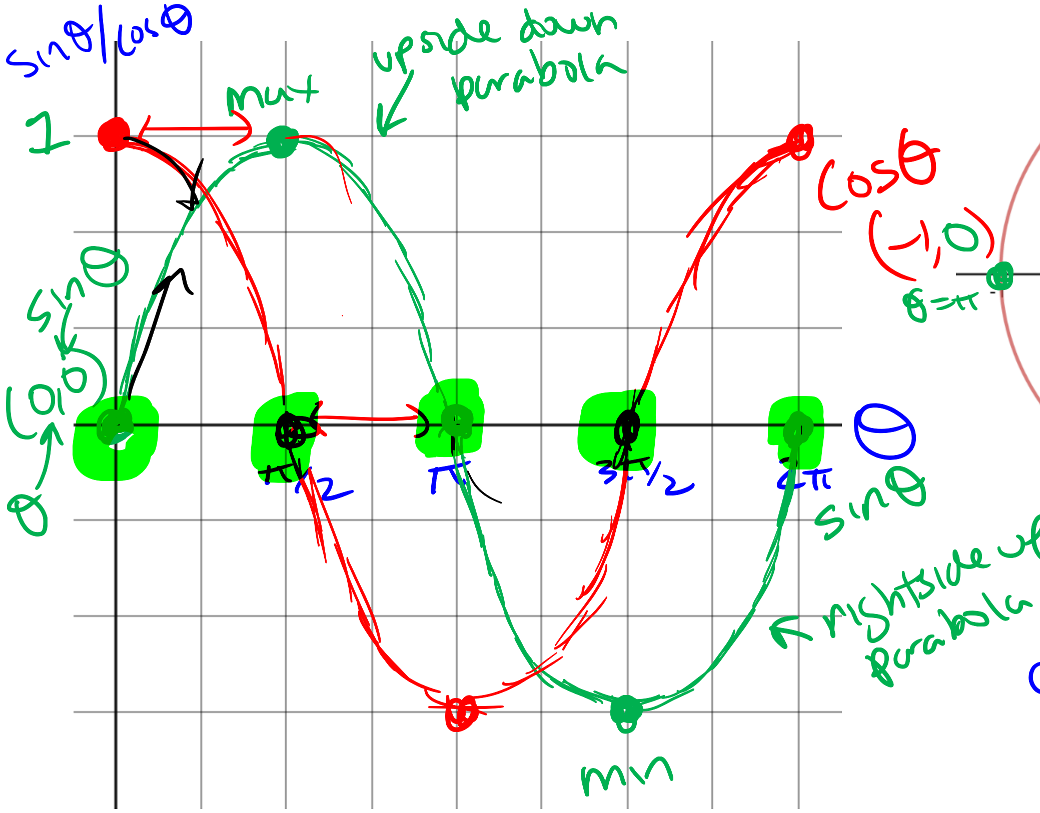
Goal:

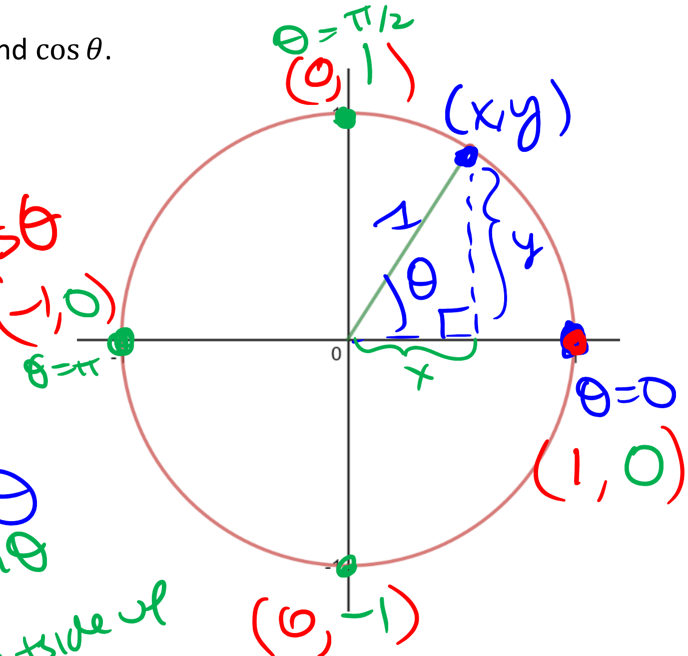
- Can quickly graph/recognize the graph of cosine and sine and manipulate their amplitudes and periods
- Knows the general form of $a\sin(bx)$ and key points along the curve.
- Can graph $\tan\theta$ and describe its basic characteristics (period, zeros, asymptotes)

Terminology:

- Amplitude
- Period
- Sinusoidal curve
- Periodic function

Using a unit circle, graph the angle θ and the values of $\sin\theta$ and $\cos\theta$.





$\cos\theta = x = \frac{adj}{hyp} = \frac{x}{1}$

$\sin\theta = y = \frac{y}{1} = y$

Characteristics

→ same max and min

→ same shape, constant shift between

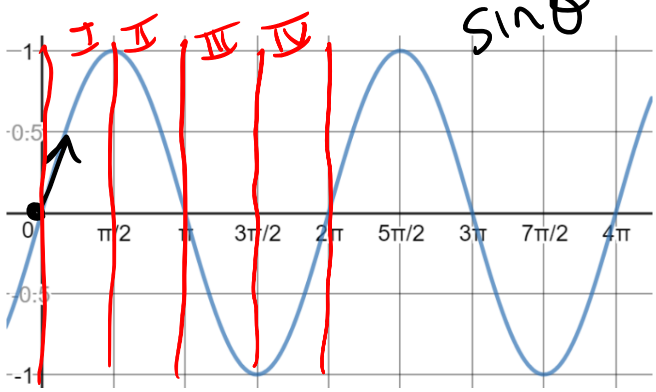
→ zeros are similar

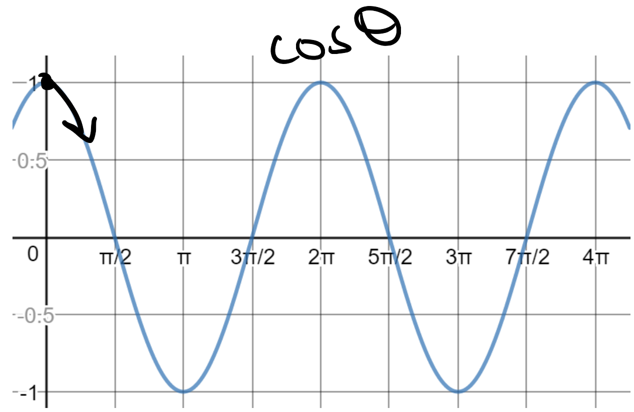
→ sine starts ↑ and cosine starts ↓

sinθ cosθ

Definition: Functions that repeat after a certain amount of time are called **periodic functions** (periodic meaning occurring at regular intervals). Periodic functions that have this "wave" shape are called **sinusoidal functions**.

The Graph of Sine and Cosine





We want to analyze this curve so that we can graph functions of the form

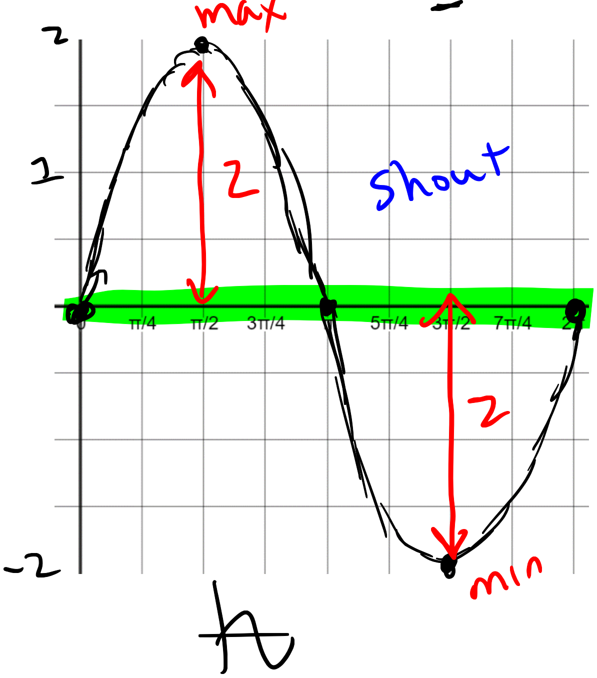
$$a \cdot \sin b\theta \quad \text{and} \quad a \cdot \cos b\theta$$

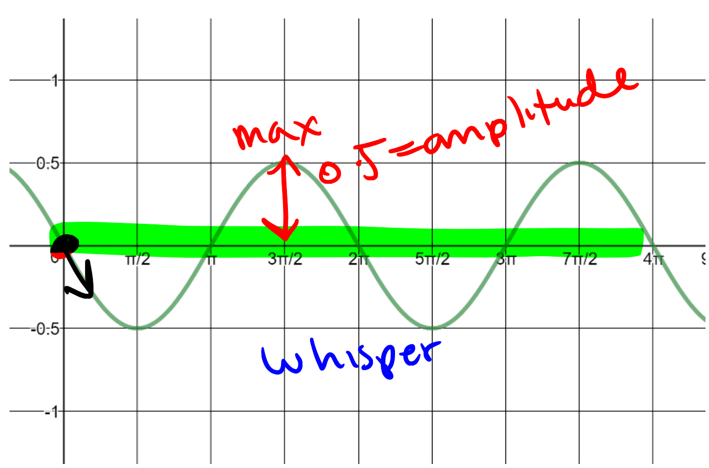
vertical stretch horizontal stretch

Definition: The *amplitude* is the distance from the midline to the maximum or minimum, or equivalently, half the distance between the max and min.

For sine and cosine, base amplitude is 1, changing a (amplitude) changes the radius of the unit circle.

Example: Graph the function $2 \sin \theta$ and determine the equation to the other graph.

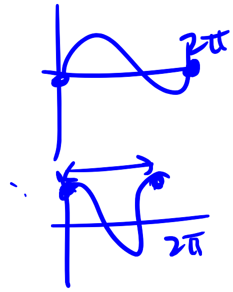




$-0.5 \sin \theta$

Definition: The *period* is the length of one complete cycle of a periodic function. Not necessarily how long it takes to repeat itself, but how long it takes to repeat the pattern.

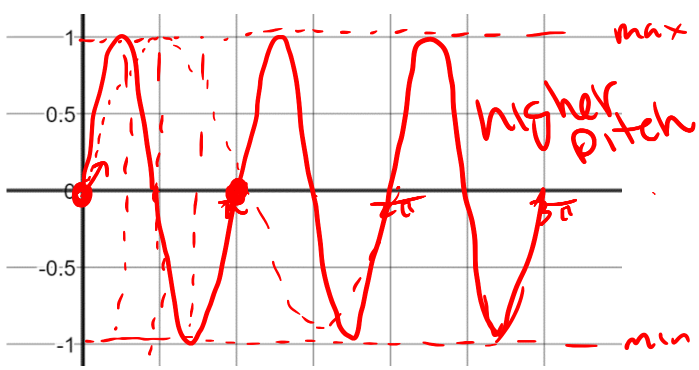
For sine and cosine, The period is $2\pi = T$

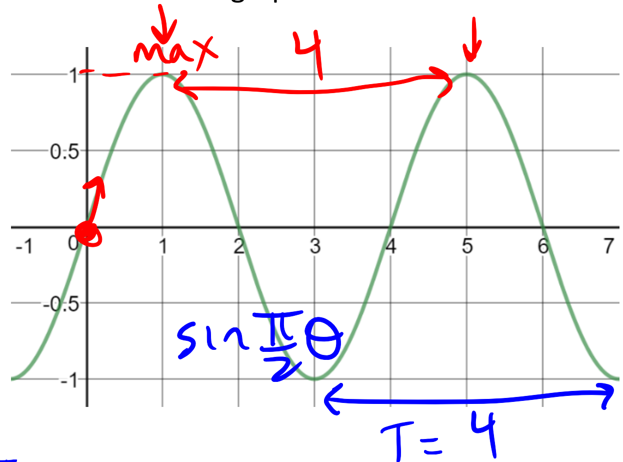


in general for $\sin b\theta$

The period, $\boxed{T = \frac{2\pi}{b}}$

Example: Graph the equation $\sin(\overset{b}{2}\theta)$ and determine the equation to the other graph

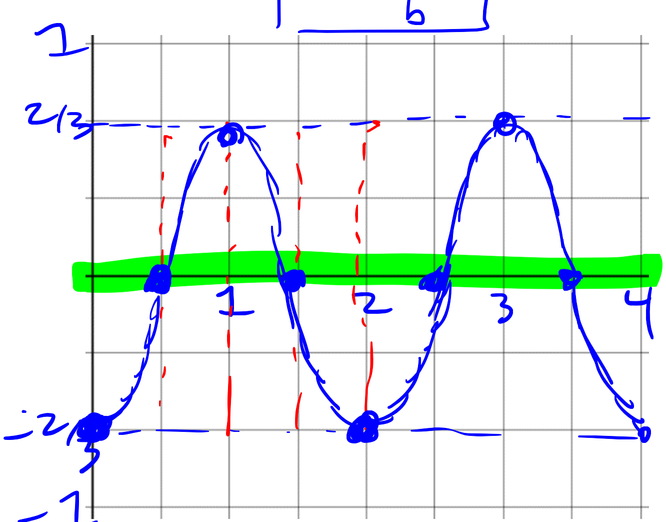


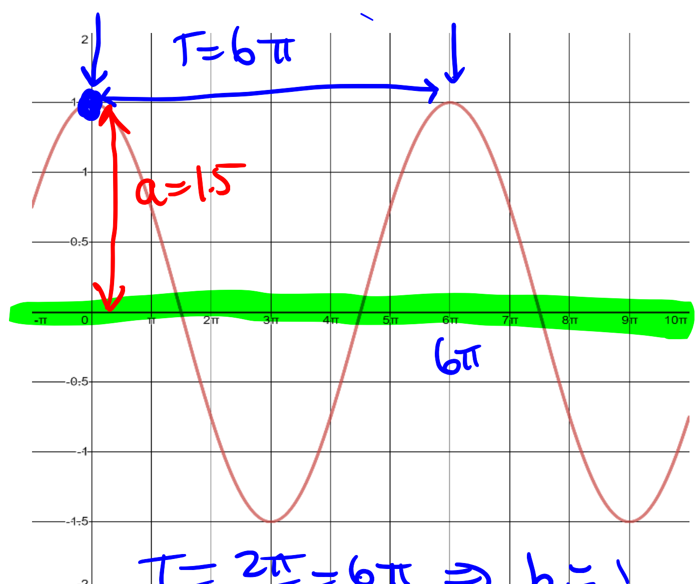


$T = \frac{2\pi}{b} = \pi$

$T = \frac{2\pi}{b} = 4 \Rightarrow b = \frac{\pi}{2}$

Practice: Graph $\boxed{-\frac{2}{3}\cos(\underset{b}{\pi}\theta)}$ and determine the equation to the other graph.





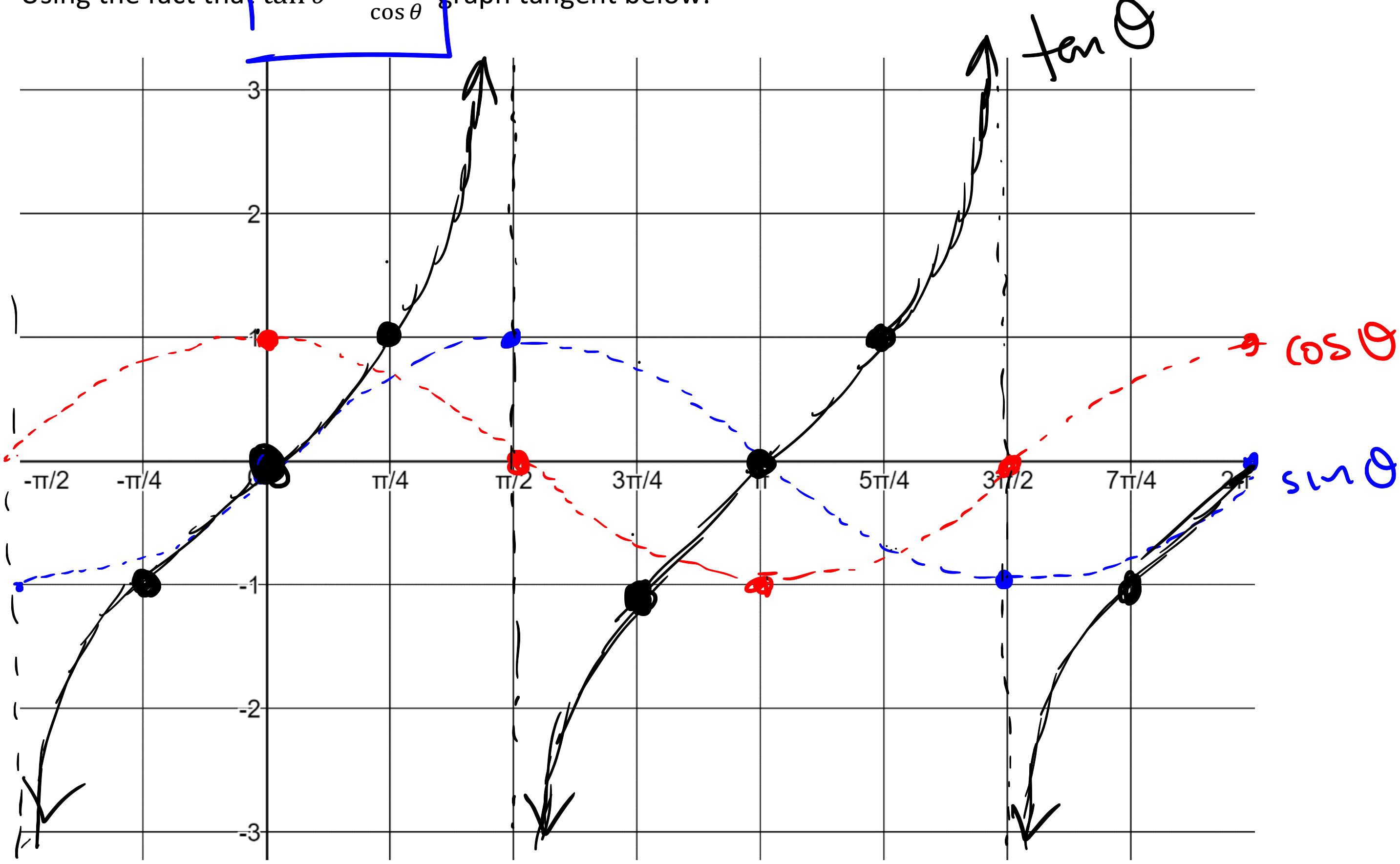
amplitude $= +\frac{2}{3}$

$T = \frac{2\pi}{b} = \frac{2\pi}{\pi} = 2$

$T = \frac{2\pi}{b} = 6\pi \Rightarrow b = \frac{1}{3}$

$1.5 \cos \frac{\theta}{3}$

Using the fact that $\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$ graph tangent below:



Characteristics

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The textbook uses this section to introduce you into building trig equations that model moving lengths of triangles (an important part of calculus called **related rates**).

Example: A baseball is hit and travels along the curve $y = x(20 - 5x)$ where y is the vertical distance and x is the horizontal distance from the ball to the batter. Relate the angle the baseball makes with the batter and the horizontal distance away from the batter.

Practice: A police light shines light in two opposite directions and makes a full revolution every 1 second. If the light is positioned 10m from a wall write an expression for the distance the light travels along the wall as a function of the angle and then as a function of time.

Suggested Practice Problems: 5.1 # 4-10, 13-15, 20 5.3 # 3, 5, 8-12
Textbook Reading: 5.1 and 5.3 page 222-231 and 256-261. Key Ideas page 232 and 262
Next Class: Transformations of Trig Function