Function Transformations: Translations

Goals:

- Describe a horizontal and vertical translations in the form $T(x) = f(x - h) + k$.
- Understands that horizontal translations look to act in the opposite direction.
- Describe a horizontal and vertical stretch/compression/reflection in the form $T(x) = a \cdot f(bx)$
- Understands that the intercepts are invariant points after an expansion or reflection.

Terminology:

- Translation
- Mapping
- Image
- Expansion & Compression
- Reflection

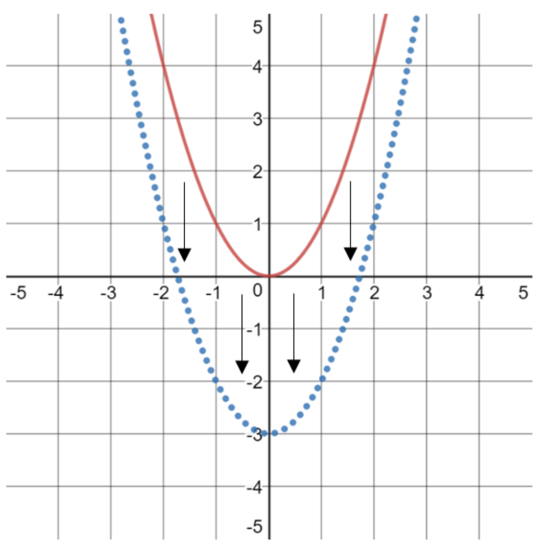
Functions are operations. When we see the function $f(x) = y$ what is being denoted is a relationship of x to y and we can write it as follows using **mapping notation**

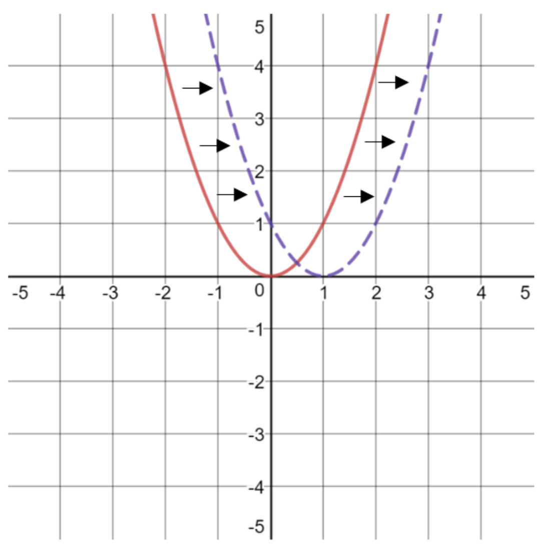
$$x \mapsto y$$

For example: If $f(1) = 3$ and $f(2) = -1$ then

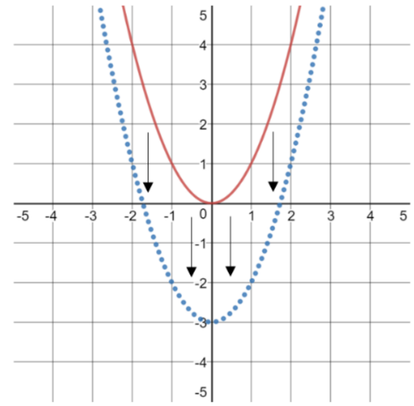
We are going to be looking at two major ways we can manipulate a function $f(x)$, and transform it into a new function, $T(x)$. For now, we will focus on just sliding the function around in 2D space (can move it horizontally and vertically). These are called **translations**.

Examples:

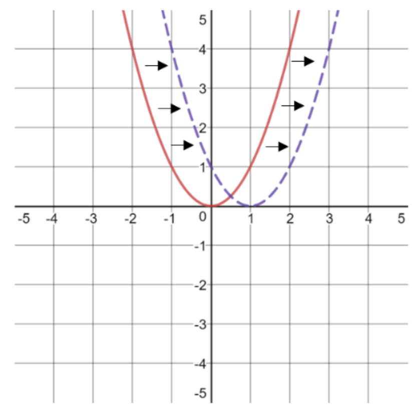




For a **vertical translation**, we take our original function where $y = f(x)$ and...

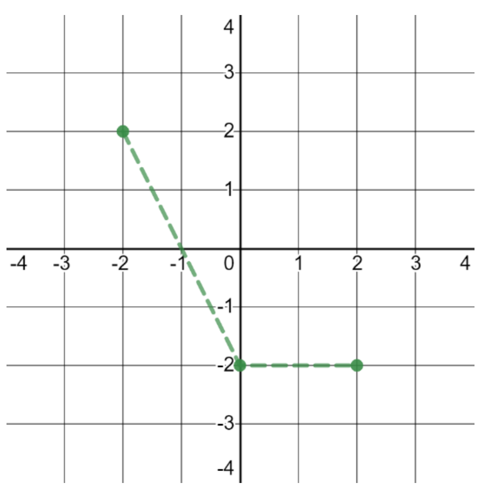


For a **horizontal translation**, we shift the function left and right but ...

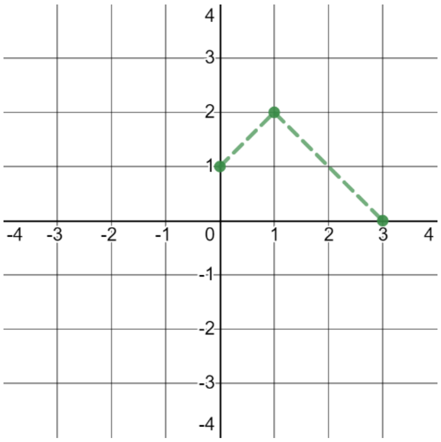


** After a transformation, the resulting function is called the **image function**.

Example 1: Given the graph of f, graph the image function after being translated 2 units right and 1 unit down. Write the mapping notation and function notation of the transformation

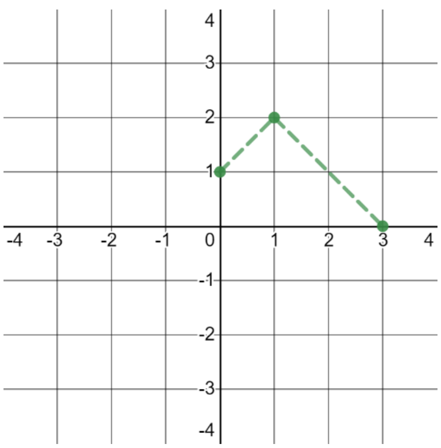


Practice: Given the graph of g, graph the image function after being translated 1 unit left and 2 units up. Write the mapping notation and function notation of the transformation



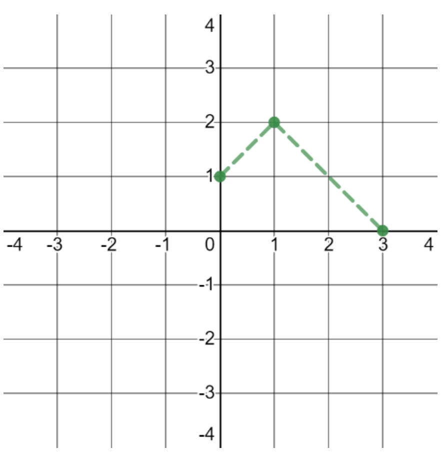
Practice: Given the graph of g, graph the image function after it has been translated as follows:

$$(x, y) \mapsto (x, y - 3)$$

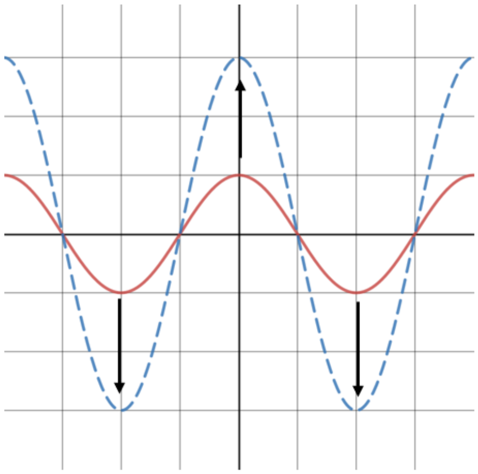


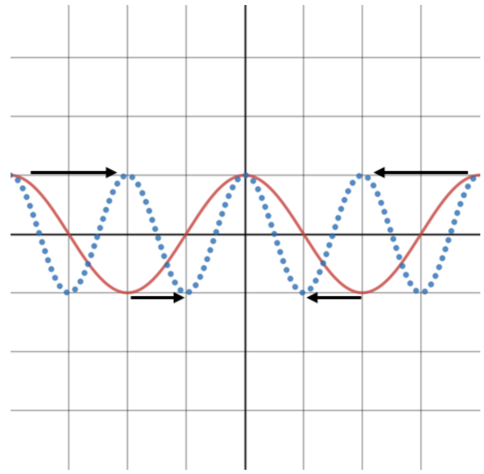
Practice: Given the graph of g, graph the image function after it has been translated as follows:

$$T(x) = g(x - 1)$$

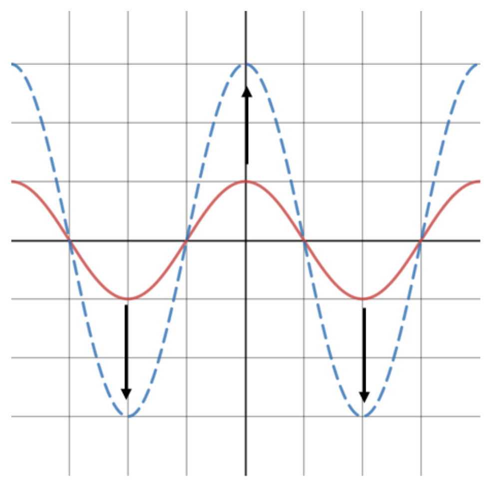


Aside from translating a function which preserves the general characteristics of the function (it just got moved around the graph) we can transform the graph in a more significant manner by stretching and compressing it relative to either axis.



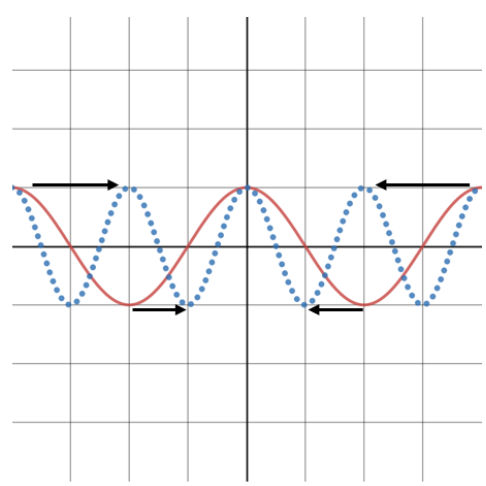


For a **vertical expansion or compression** (expansion about the x-axis), we take our original function where $y = f(x)$ and...



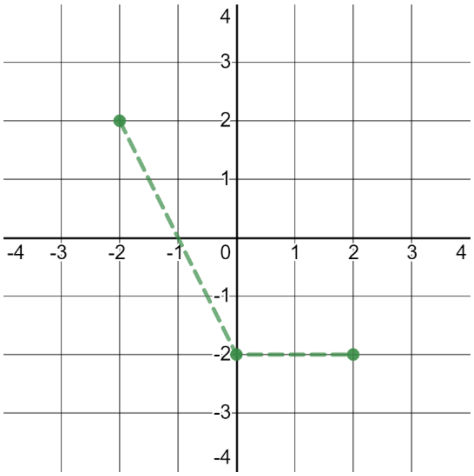
** $(x, 0) \mapsto$

For a **horizontal expansion or compression** (expansion about the y-axis), we have that...



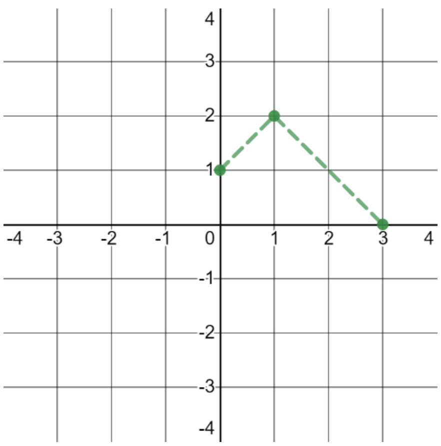
** $(0, y) \mapsto$

Example 2: Given the graph of f, graph the image function after it has vertically been compressed by a factor of 2 and horizontally expanded by a factor of $\frac{3}{2}$. Write the mapping notation and function notation of the transformation.



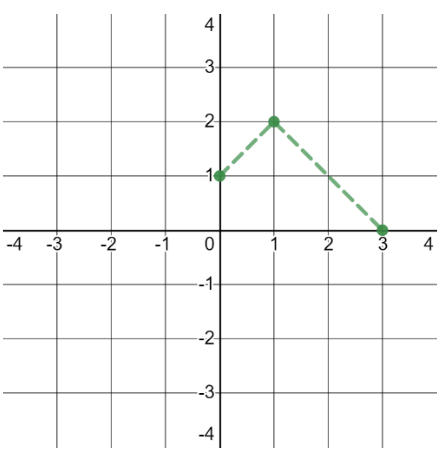
Practice: Given the graph of g, graph the image function after it has been translated as follows:

$$(x, y) \mapsto \left(\frac{1}{3}x, y\right)$$



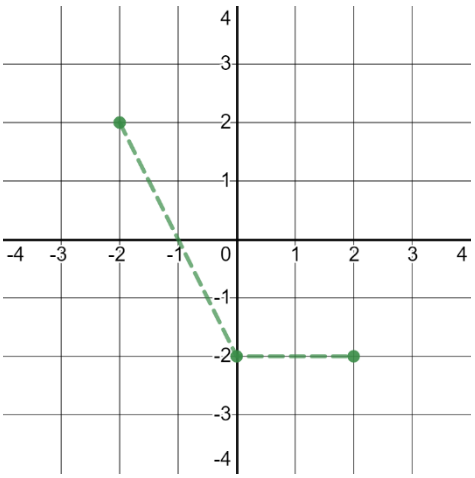
Practice: Given the graph of g, graph the image function after it has been translated as follows:

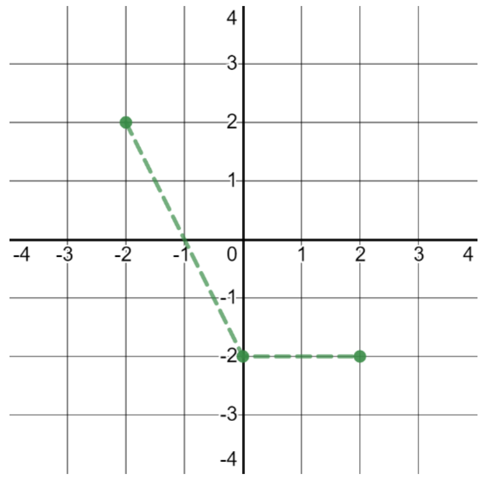
$$T(x) = 2g\left(\frac{1}{2}x\right)$$



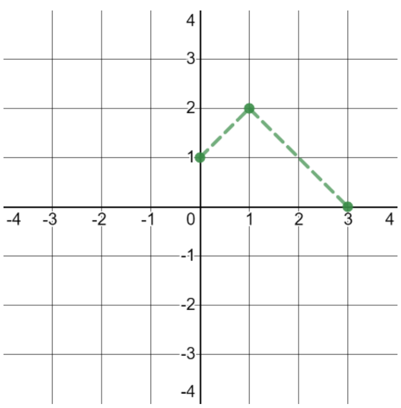
If the value of a or b is negative, this means we have the cases of a **reflection**.

$$(x, y) \mapsto (x, -y) \qquad\qquad (x, y) \mapsto (-x, y)$$



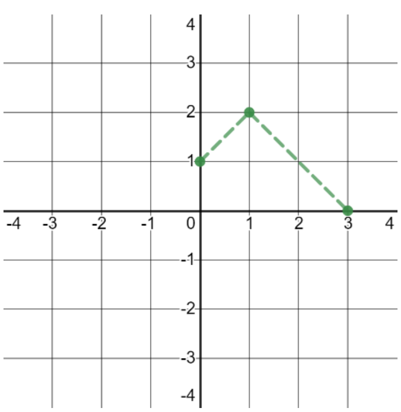


Practice: Given the graph of g, graph the image function after it has been reflected over the y-axis. Write the mapping notation and function notation of the transformation.



Practice: Given the graph of g, graph the image function after it has been translated as follows:

$$T(x) = -\frac{3}{2}g(-x)$$



Suggested problems: 1.1 page 12 – 14 # 2-4, 8-12, 16, 18, 19, C1 1.2 page 28 – 31 # 3-5, 7, 10, 12, 14, 16, C1, C2, C3	
Textbook Reading: 1.1 page 6-12 & 1.2 page 16-27 **Key Ideas** on page 12 and 27	
Next Class: Combining transformations and identifying transformed graphs	