Function Transformations: Translations

Goals:

- Describe a horizontal and vertical translations in the form $T(x) = f(x-h) + k$.
- Understands that horizontal translations look to act in the opposite direction.
- Describe a horizontal and vertical stretch/compression/reflection in the form $T(x) = a \cdot f(bx)$
- Understands that the intercepts are invariant points after an expansion or reflection.

Terminology:

- Translation
- Mapping
- Image
- Expansion & Compression
- Reflection

input

$f(x) = \sqrt{x}$; $f(x) = x^2$; $\frac{1}{x}$

output

Functions are operations. When we see the function $f(x) = y$ what is being denoted is a relationship of x to y and we can write it as follows using **mapping notation**

$$x \mapsto y$$

Domain → Range

$4 \mapsto 2$ $\sqrt{x}$

$9 \mapsto 81$ x^2

For example: If $f(1) = 3$ and $f(2) = -1$ then

$1 \mapsto 3$ $(1,3)$

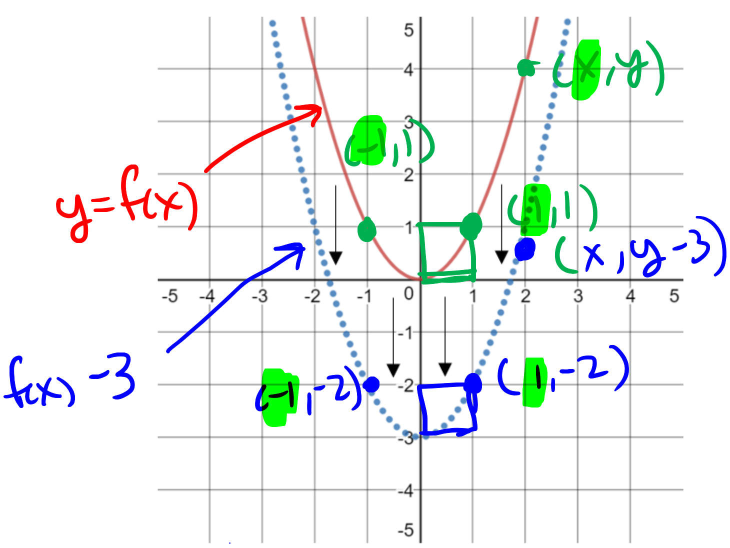
$2 \mapsto -1$ $(2,-1)$

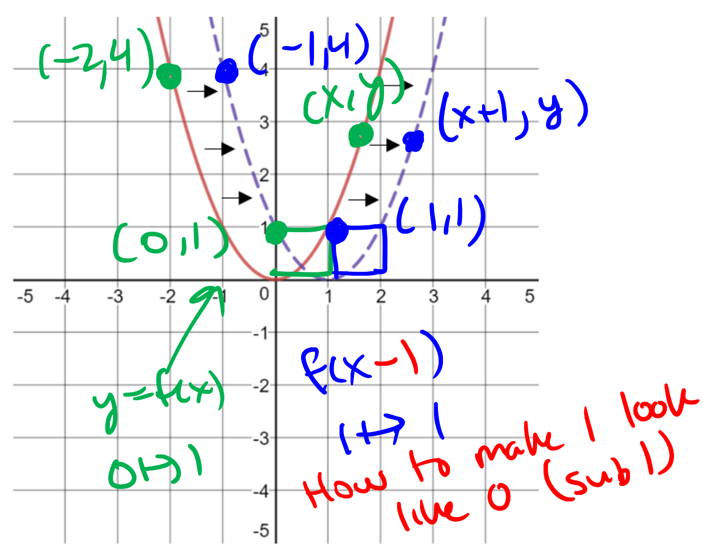
$\sqrt{1+4} = \sqrt{1} + \sqrt{4}$? $f(a+b) \neq f(a) + f(b)$

We are going to be looking at two major ways we can manipulate a function $f(x)$, and transform it into a new function, $T(x)$. For now, we will focus on just sliding the function around in 2D space (can move it horizontally and vertically). These are called **translations**.

Shift $(x,y) \mapsto (u,v)$

Examples:





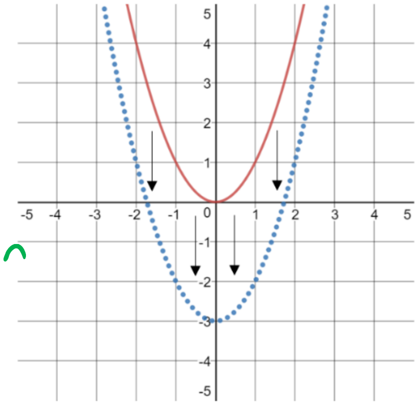
$(x,y) \mapsto (x, y+d)$ shift up/down

$g(x) = f(x) + d$

$(x,y) \mapsto (x+c, y)$ shift left/right

$g(x) = f(x-c)$

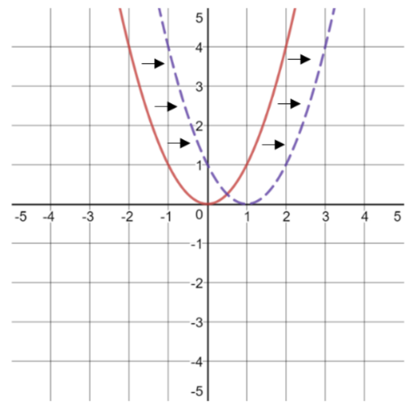
For a **vertical translation**, we take our original function where $y = f(x)$ and...



$(x, y) \mapsto (x, y+d)$ mapping

$g(x) = f(x) + d$ function notation

For a **horizontal translation**, we shift the function left and right but ...



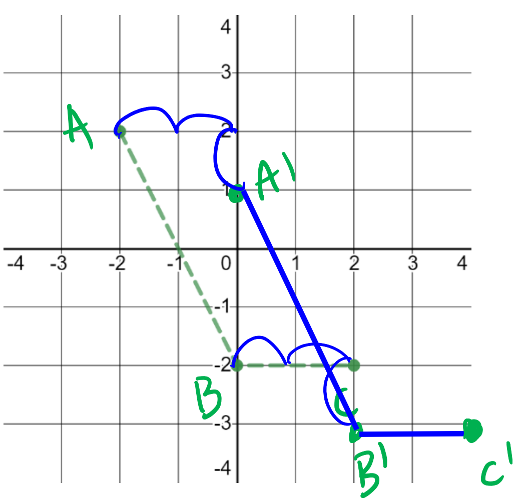
$(x, y) \mapsto (x+c, y)$

$g(x) = f(x-c)$

✲ shifts left/right look opposite (in function form)

** After a transformation, the resulting function is called the **image function**.

Example 1: Given the graph of f, graph the image function after being translated 2 units right and 1 unit down. Write the mapping notation and function notation of the transformation



$(x, y) \mapsto (x+2, y-1)$

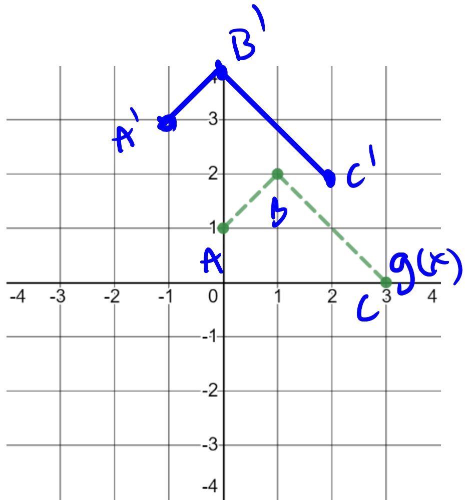
$(-2, 2) \mapsto (0, 1)$

$(0, -2) \mapsto (2, -3)$

$(2, -2) \mapsto (4, -3)$ ✓

$g(x) = f(x-2) - 1$

Practice: Given the graph of g, graph the image function after being translated 1 unit left and 2 units up. Write the mapping notation and function notation of the transformation



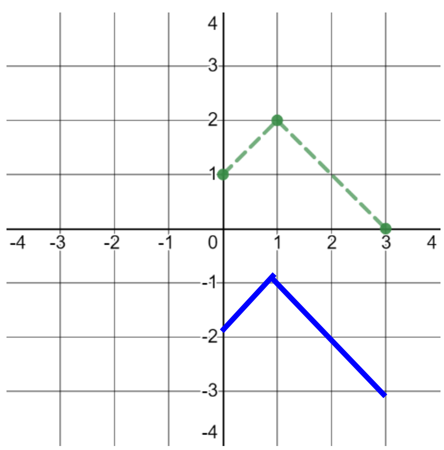
$(x,y) \mapsto (x-1, y+2)$

$h(x) = g(x+1) + 2$

Practice: Given the graph of g, graph the image function after it has been translated as follows:

$$(x, y) \mapsto (x, y-3)$$

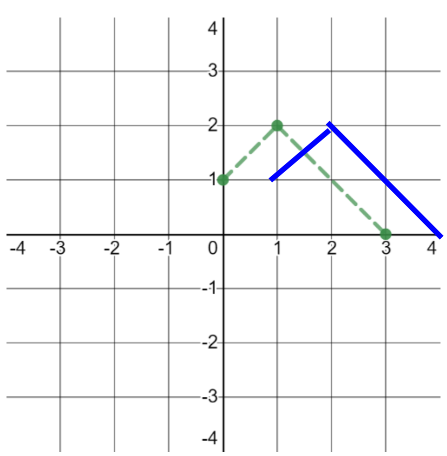
shft down 3



$h(x) = g(x) - 3$

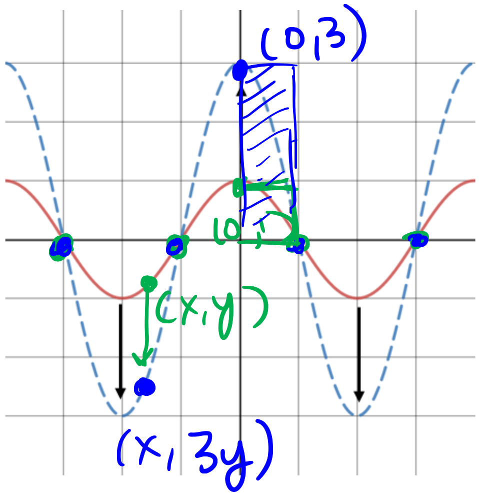
Practice: Given the graph of g, graph the image function after it has been translated as follows:

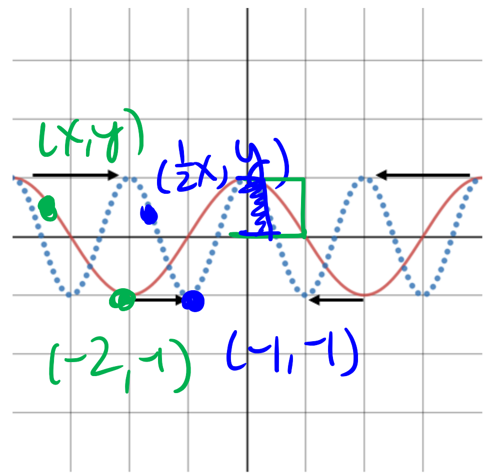
$$T(x) = g(x-1)$$



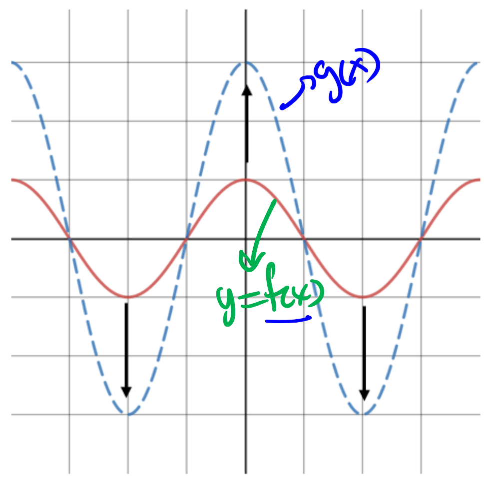
$(x,y) \mapsto (x+1, y)$

Aside from translating a function which preserves the general characteristics of the function (it just got moved around the graph) we can transform the graph in a more significant manner by stretching and compressing it relative to either axis.





For a **vertical expansion or compression** (expansion about the x-axis), we take our original function where $y = f(x)$ and…



$(x,y) \mapsto (x, 3y)$ >1 expand

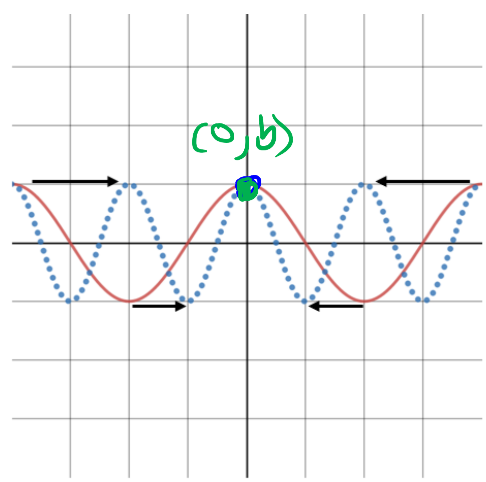
$(x,y) \mapsto (x, a\cdot y)$ stretch vertical

$g(x) = a\cdot f(x)$

** $(x, 0) \mapsto$ $(x, 3\cdot 0) = (x, 0)$ invarient point

x-int

For a **horizontal expansion or compression** (expansion about the y-axis), we have that…



<1 compress

$(x,y) \mapsto (\frac{1}{2}x, y)$

$(x,y) \mapsto (b\cdot x, y)$

$y_2 = g(x) = f(bx)$ here when

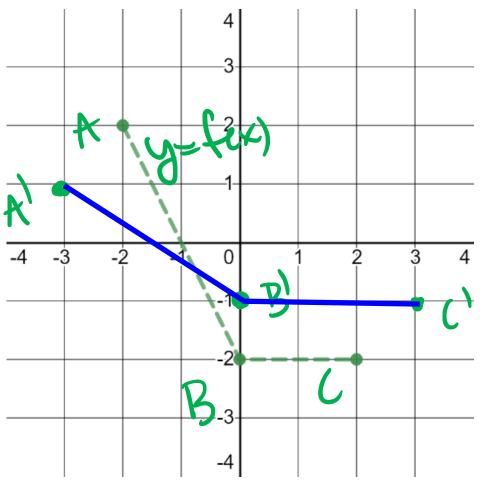
$b<1 \Rightarrow$ expan.

$b>1 \Rightarrow$ comp.

** $(0, y) \mapsto$ $(\frac{1}{2}\cdot 0, y) = (0, y)$ invarient point

y-int

Example 2: Given the graph of f, graph the image function after it has vertically been compressed by a factor of 2 and horizontally expanded by a factor of $\frac{3}{2}$. Write the mapping notation and function notation of the transformation.



$(x,y) \mapsto \left(\frac{3}{2}x, \frac{1}{2}y\right)$

$(-2,2) \mapsto (-3,1)$

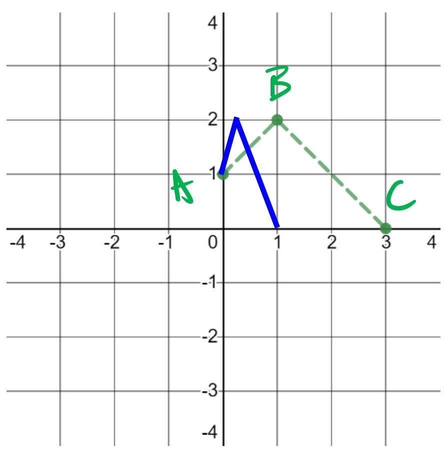
$(0,-2) \mapsto (0,-1)$

$(2,-2) \mapsto (3,-1)$

$g(x) = \frac{1}{2} f\left(\frac{2}{3}x\right)$

Practice: Given the graph of g, graph the image function after it has been translated as follows:

$$(x,y) \mapsto \left(\frac{1}{3}x, y\right)$$



$(0,1) \mapsto (0,1)$

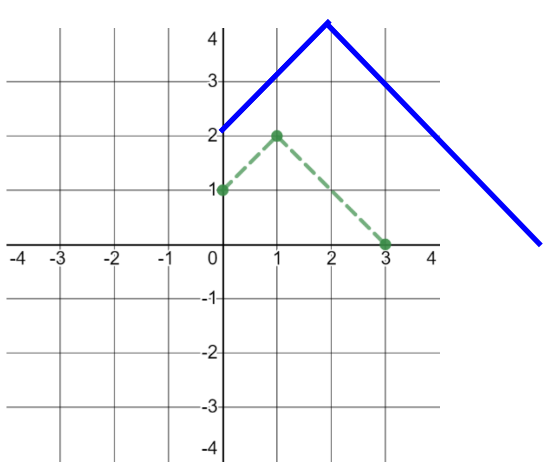
$(1,2) \mapsto \left(\frac{1}{3},2\right)$

$(3,0) \mapsto (1,0)$

$h(x) = g(3x)$

Practice: Given the graph of g, graph the image function after it has been translated as follows:

$$T(x) = 2g\left(\frac{1}{2}x\right)$$



$(x,y) \mapsto (2x, 2y)$

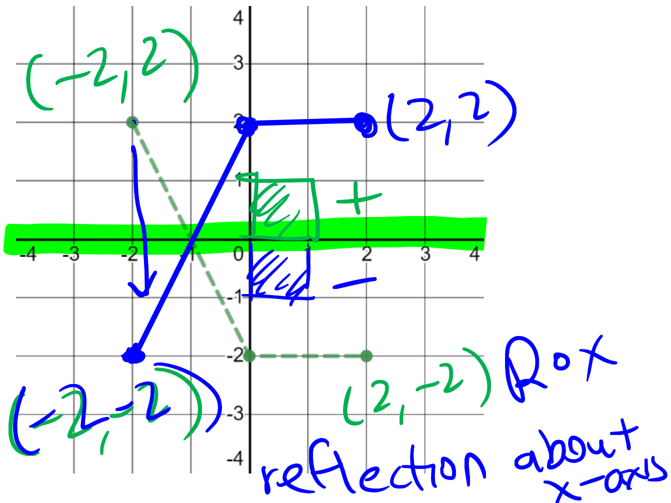
$(1,0) \mapsto (2,0)$

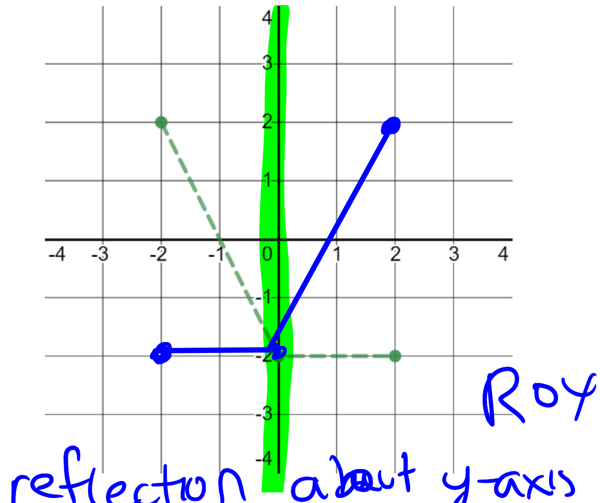
$(1,2) \mapsto (2,4)$

$(3,0) \mapsto (6,0)$

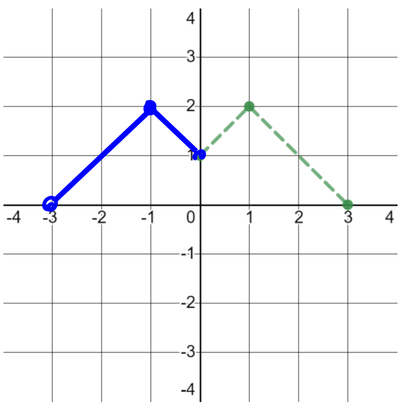
If the value of a or b is negative, this means we have the cases of a **reflection**.

$(x, y) \mapsto (x, -y)$ $(x, y) \mapsto (-x, y)$





Practice: Given the graph of g, graph the image function after it has been reflected over the y-axis. Write the mapping notation and function notation of the transformation.

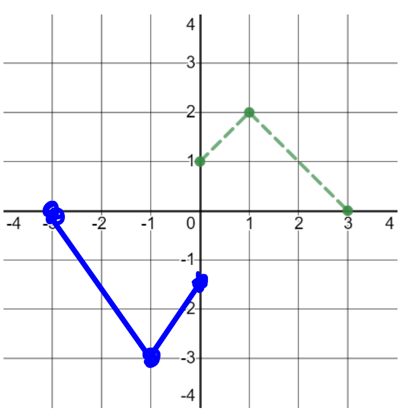


$(x, y) \mapsto (-x, y)$

$T(x) = g(-x)$

Practice: Given the graph of g, graph the image function after it has been translated as follows:

$$T(x) = -\frac{3}{2}g(-x)$$



$(x, y) \mapsto (-x, -\frac{3}{2}y)$

$(0, 1) \mapsto (0, -3/2)$

$(1, 2) \mapsto (-1, -3)$

$(3, 0) \mapsto (-3, 0)$

Suggested problems: 1.1 page 12 – 14 # 2-4, 8-12, 16, 18, 19, C1 1.2 page 28 – 31 # 3-5, 7, 10, 12, 14, 16, C1, C2, C3	
Textbook Reading: 1.1 page 6-12 & 1.2 page 16-27 **Key Ideas** on page 12 and 27	
Next Class: Combining transformations and identifying transformed graphs	