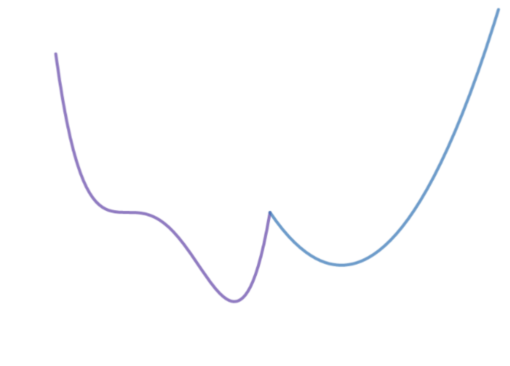
First and Second Derivative Test

Goal:
- Can use the first derivative test and second derivative test to find extrema
- Can determine concavity by considering f' or f''

Terminology:
- First Derivative Test
- Concavity
- Inflection Point
- Second Derivative Test

We are going to consider that $f: [a, b] \to \mathbb{R}$ is continuous and we want to find a way to determine if a critical point is a maximum or a minimum. Note that f need not be differentiable on (a, b).

x	a	c_1	c_2	c_3	c_4	b
$f'(x)$						
Behaviour of f						



Recall: If f is differentiable on (a, b), continuous on $[a, b]$ and $f'(x) > 0\ \forall\ x \in (a, b)$ then f is (strictly) increasing on $[a, b]$.

Proof:

Theorem: The first derivative test says that if f is continuous on $[a, b]$ and we have that $f'(x) > 0$ on (a, c) and $f'(x) < 0$ on (c, b), then $x = c$ is a local maximum.

Proof:

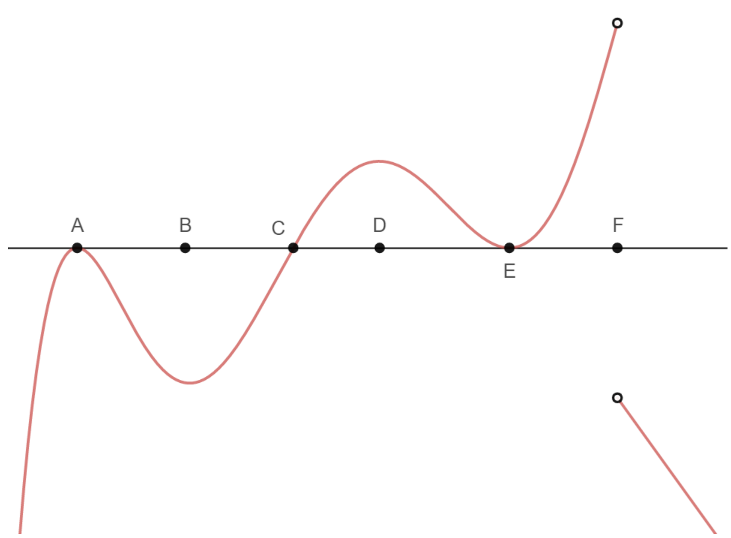
Practice: Use the first derivative test to find all extrema of the following functions:

$f(x) = x^3 - x, \qquad x \in [-1, 2]$

$g(x) = 2x^4 + 4x^3 - 1, \qquad x \in [-2, 1]$

$h(x) = e^{\cos x}, \qquad x \in [-2, 4]$

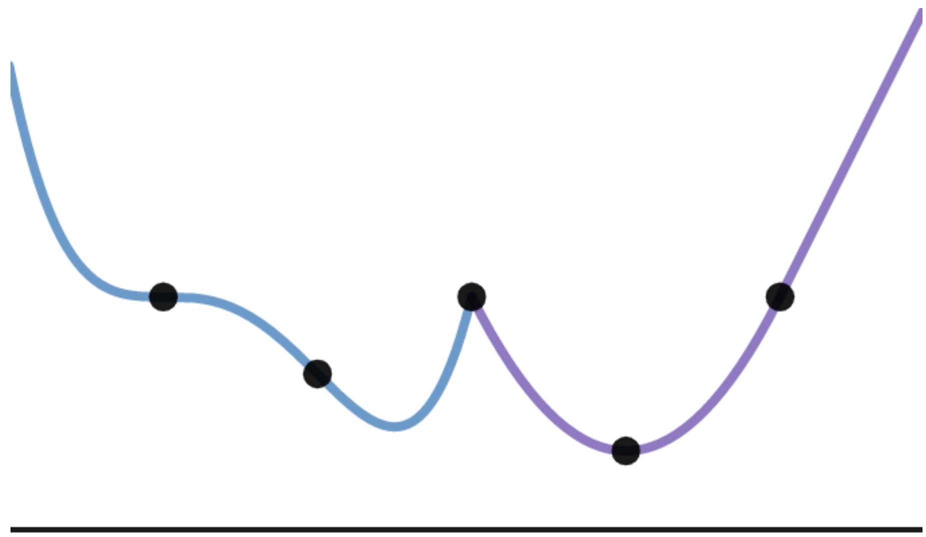
The following is a graph of $\frac{dk}{dx}$ for some continuous k



Definition: A differentiable function is *concave up* on (a, b) if f' is (strictly) increasing on (a, b). Note the open intervals as f'may not exist at the endpoints.

**If f is differentiable on $[a, b]$ then concave up implies f' is (strictly) increading on $[a, b]$

Note that if the function is twice differentiable, we can say that if f' is increasing on $[a, b]$, then $f''(x) > 0 \; \forall \; x \in (a, b)$



Theorem: The second derivative test says that if f is twice differentiable in an open interval (a, b), and $f'(c) = 0$, and $f''(x) < 0 \; \forall \; x \in (a, b)$, then $f(c)$ is a local maximum on $[a, b]$.

Proof:

Definition: Whenever f'' changes sign, we have a *inflection point*

Practice: Determine the intervals the functions are concave up/down

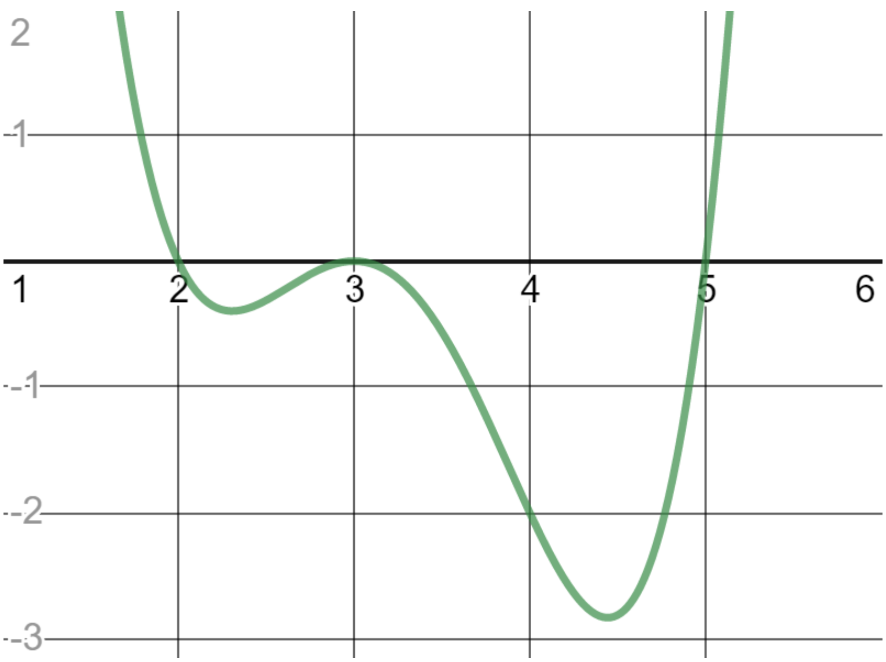
$$\ell(x) = \frac{x}{x^2 + 1}$$

$$m(x) = x^3 e^x$$

Practice: Use the second derivative test to find the local max and minimums. Assume that n is continuous.

x	$x < A$	A	$A < x < B$	B	$B < x < C$	C	$x > C$
$n''(x)$	Positive	0	Negative	undefined	Positive	0	Positive
$n'(x)$	0 For some $a < A$	5	0 For some $a_b \in (A, B)$	undefined	0 For some $b_c \in (B, C)$	0	Positive

Consider the graph of p'' below. If $p'(x) = 0$ when $x = 1.7, 4,$ and 5.3 then determine where the extrema of p occur and the type of extrema.



Practice Problems: 4.3: # 5-12 (select), 13-28 (select and use technology), 41-44, 50, 52	
Textbook Readings: 4.3 page 194-200	
Workbook Practice: page 182-193	
Next Day: Curve Sketching	