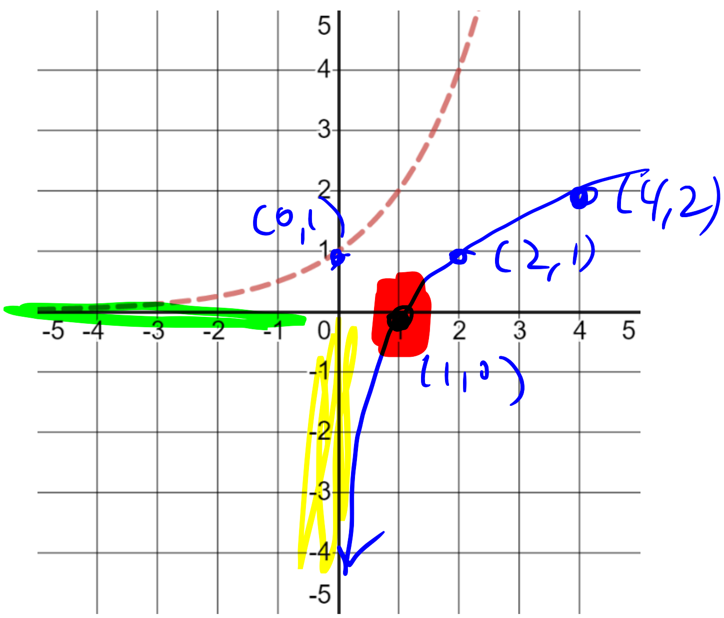
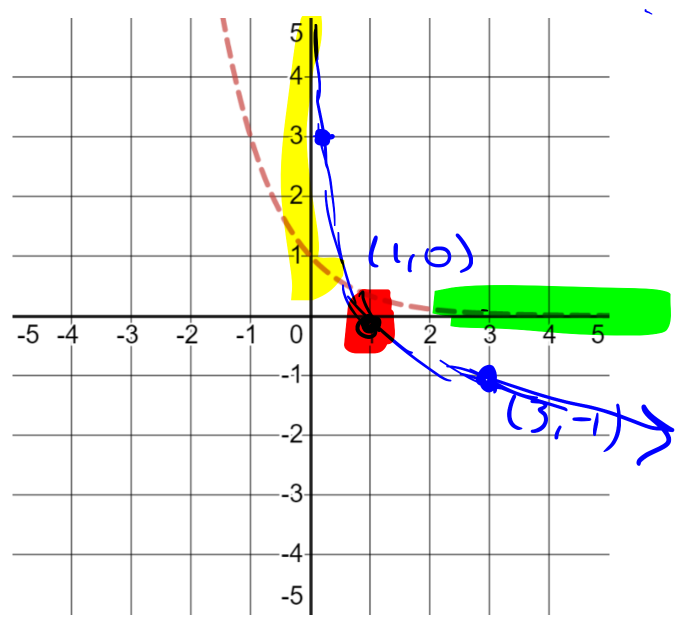
Logarithmic Functions

Goal: • Can graph the inverse of the exponential and determine the equation of the inverse using logs. • Can graph log functions by finding the exponential inverse and using that. • Can evaluate basic logs. • Can graph functions of the form $f(x) = a \cdot \log\big(b(x-c)\big) + d$ and use the graph to solve applications.
Terminology: • Logarithm • Natural Logarithm • Exponential Form • Logarithmic Form

Graph the inverse of $y = 2^x$ and $y = (1/3)^x$





☆ These have vertical asymptotes.

since (0,1) was invariant between exponentials we think (1,0) will be special for the inverse

Since the exponential passes its horizontal line tests, it is one-to-one and therefore the inverse is a function.

$$y = B^x = f(x)$$

f^{-1} exists

$f^{-1}(B^x = f(x))$ $\rightarrow$ $x = B^y$

$f^{-1}(B^x) = x$

$\log_B(x) = \cancel{\log_B} B^y$

This $\log_B = f^{-1} \Rightarrow \log_B x = y = f^{-1}(x)$

There are three common bases that you will use depending on your field.

- Engineering: Base 10 $\log_{10} x = \log x$ common log
- Science and Mathematics: Base e $\log_e x = \ln x$ natural log $\rightarrow$ $\log x$
- Computer Science: Base 2 $\log_2 x = lb\, x$ binary log $\rightarrow$ $\log x$

Example: Solve for k

$\log(500 = \cancel{10}^k)$ $k = 2$, $k = 3$

$\log 500 = k$

$2.69897\ldots = k$

$\ln(2 = \cancel{e}^k)$ $e = 2.7\ldots$ $0, 1$

$\ln 2 = k$

$0.693147\ldots = k$

$2 = e^{0.7} = e^{\ln 2}$

Practice: Solve for x

$1200 = 10^x$

$\log 1200 = x$

$3.079\ldots = x$

$20 = e^x$

$\ln 20 = x$

$2.9957\ldots = x$

$3 = \cancel{\log_2} x$

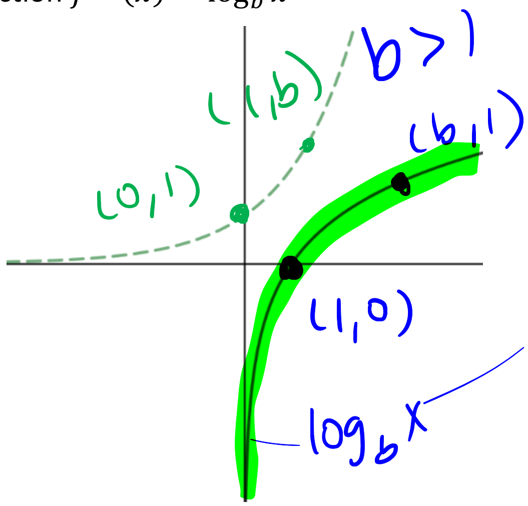
$2^3 = x$

$8 = x$

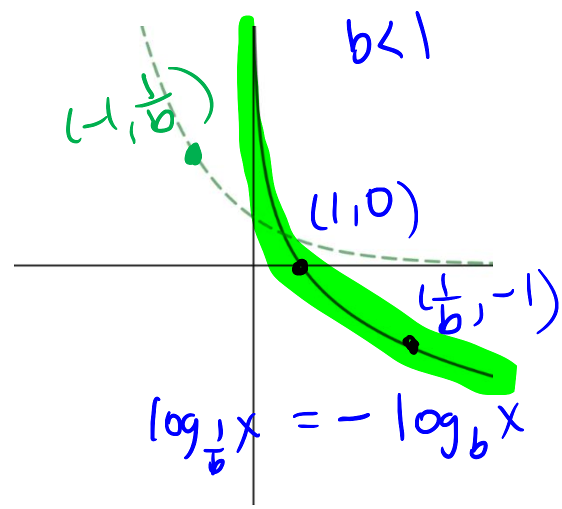
$8 = \cancel{\ln} x$

$e^8 = x$

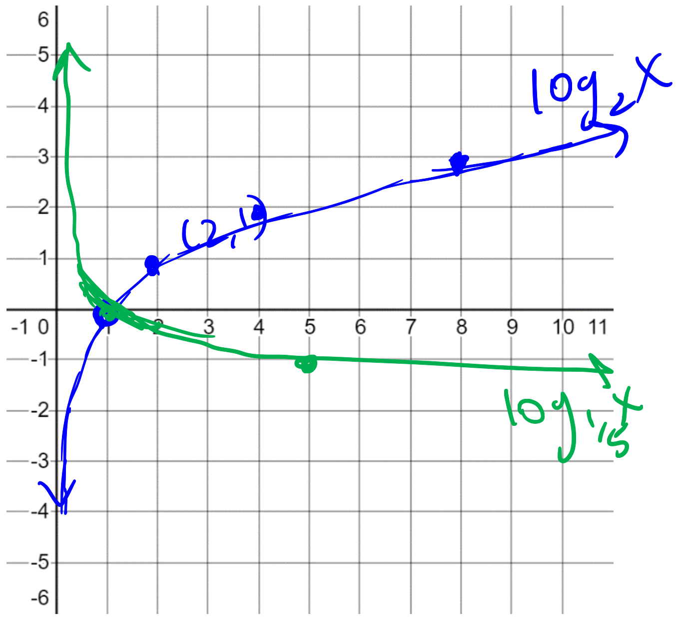
Since the exponential function $f(x) = b^x$ needs that the base $b > 0$ and $b \neq 1$, we have the same restriction on the function $f^{-1}(x) = \log_b x$

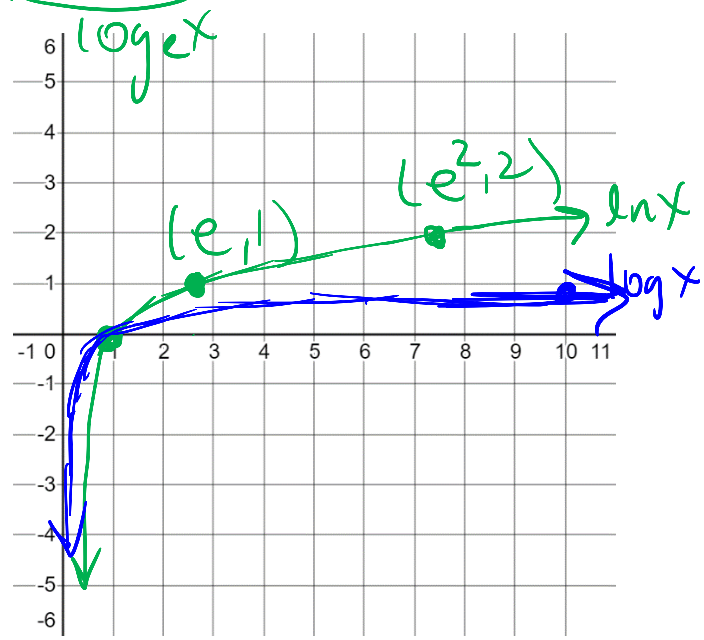


R o X

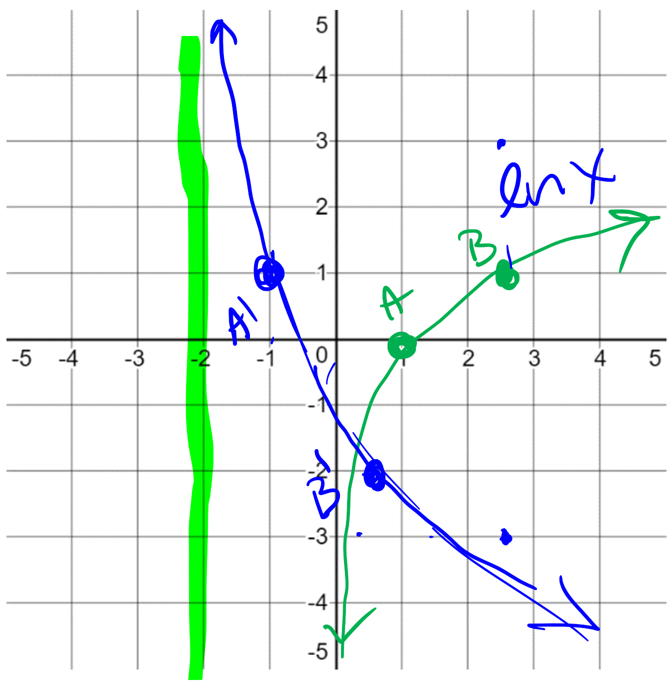


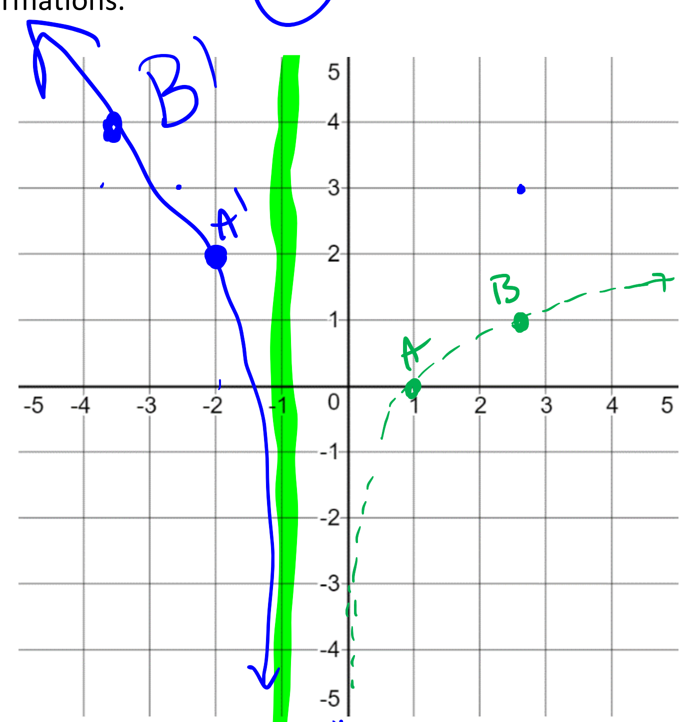
Practice: Graph the functions $\boxed{f(x) = \log_2 x}$; $g(x) = \log_{\frac{1}{5}} x$; $h(x) = \ln x$; $k(x) = \log x$





Practice: Graph the function $f(x) = \ln x$ and then graph the functions $g(x) = -3\ln(x+2) + 1$ and $h(x) = 2\ln(-x-1) + 2$ on the same grid using transformations.





$-3\ln(x+2)+1$

R0X vert. stretch by 3

left 2 up 1

$2\ln(-(x+1))+2$

vert stretch by 2

R0y

left 1 up 2

We'll practice creating log equations in later classes, but right now we want to be able to use log equations and solve simple exponential ones.

Example: In the States, the box office performance of Avengers: Endgame followed a logarithmic growth according to the equation:

left

$$B(d) = 140\ln(d+1) + 300$$

Where $B(d)$ is the total box office performance (in millions) and d is the number of days since release. What was the box office performance 2 weeks after it was released? How long after it released did it reach \$750 million in the box office?

$B(14) = 140\ln(14+1) + 300$

$= 679$ \$ million

$750 = 140\ln(d+1) + 300$

$-300 \qquad -300$

$\frac{450}{140} = \frac{140}{140}\ln(d+1)$

$e^{(450/140)}$

$e^{\frac{450}{140}} = e^{\ln(d+1)}$

$24.9 = d+1$

$d = 24$ days

Practice: The apparent magnitude of a cosmic object (compared to Vega), m, uses the intensities of the star, I, (in μW/m^2) when viewed from Earth as follows:

$$m = -2.5\log(I) - 4$$

What is the magnitude of a Venus which has an intensity of 2.2 μW/m^2? What is the intensity of Mars which has an apparent magnitude of -3? ~~Graph this function.~~

$m = -2.5\log(2.2) - 4$

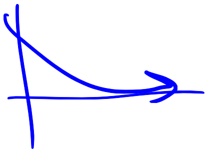
$= -4.8$ apparent mag.

$-3 = -2.5\log(I) - 4$

$10^{\frac{1}{-2.5}} = 10^{\log I}$

$10^{\frac{1}{-2.5}} = I = 0.40\,\mu W/m^2$

Practice: A cars battery drains exponentially while it is turned off. You leave your car parked over winter starting with 100% charged battery. You check 2 weeks later and the charge is 60%. Make an exponenital equation with base e for the battery charge after t days and determine when the charge will be at 10%.



$T = 14$ days

$b(t) = 100 \cdot (0.6)^{t/14}$

$= 100\left(e^{-0.511}\right)^{t/14}$

$\ln\left(0.6 = e^{k}\right)$

$\ln 0.6 = k$

$-0.511 = k$

$10 = 100\left(e^{\frac{-0.511}{14} \cdot t}\right)$

$\ln\left(0.1 = e^{\frac{-0.511}{14} t}\right)$

$\ln 0.1 = \frac{-0.511}{14} t$

$t = 63$ days.

Practice: A population of fish in a lake has had a predetor removed from the lake and the current population of 1000 can grow exponentially to a theoretical limit of 1400 based on available space and food. One month after the predator was removed the population is 1100. Make an equation with base 10 for the number of fish in the lake after t months and determine when the population of fish will be 1300.

Suggested Practice Problems: 8.1 page 380-381 # 2-4, 7-9, 12-14, 17, 18, 21, 23 8.2 page 390-391 # 4, 7, 9, 10, 13, 14, 17, C1, C3	
Textbook Reading: 8.1 and 8.2 page 372-378 and 383-388 Key Ideas on page 379 and 389	
Next Class: Log Laws	

Exponential Growth

Graphing Logarithmic Functions