

Combining Stretches and Translations

Goal:

- Describe a complete transformation in the form $T(x) = a \cdot f(b(x - c)) + d$.
- Understands why the standard order is Stretch then Translate, and how changing the order can change the image function.
- Knows the shape of core function: x^2 ; $|x|$; $\frac{1}{x}$

Terminology:

- none

There is nothing stopping us from doing a shift and stretch in tandem; however, we need to be mindful of the order.

When we say: “Perform a vertical expansion by a factor of 2, and then shift it up 2 units”, we really mean

$$(x, y) \mapsto$$

But when we say: “Shift it up 2 units and then expand it vertically by a factor of 2”, we are doing

$$(x, y) \mapsto$$

**** When combining transformations, the order we apply it is important!**

In function notation, the standard way of expressing a combination of transformations is:

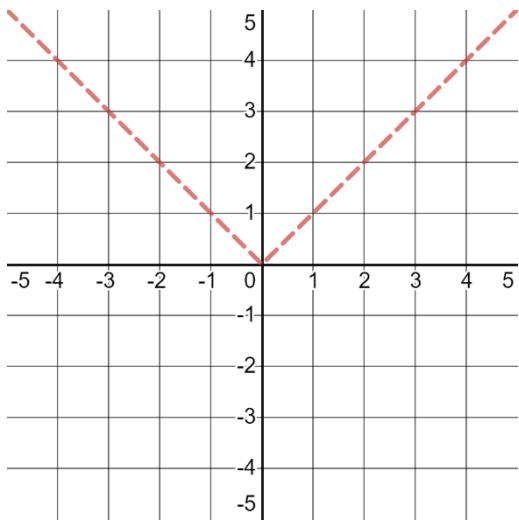
$$T(x) = a \cdot f(b(x - c)) + d$$

Which translates in mapping notation to:

$$(x, y) \mapsto$$

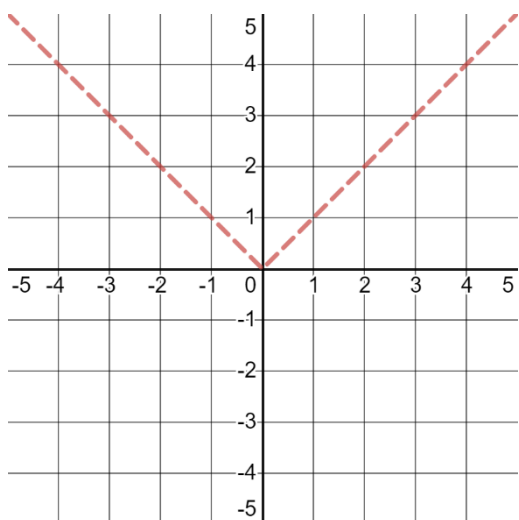
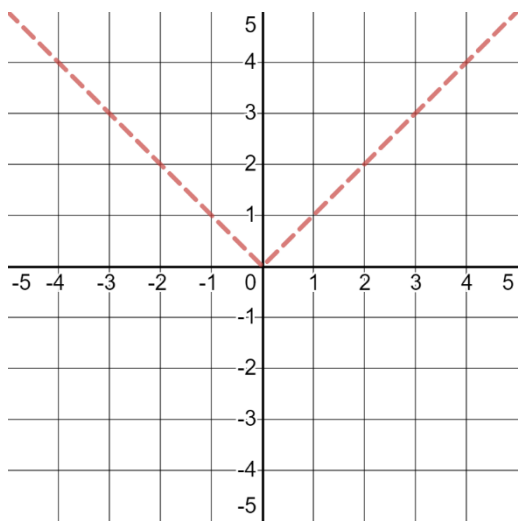
Example 1: Given that $f(x) = |x|$, sketch the image of the following and write an equation for the image that uses absolute value instead of f

$$-f(0.5(x + 2)) + 1$$



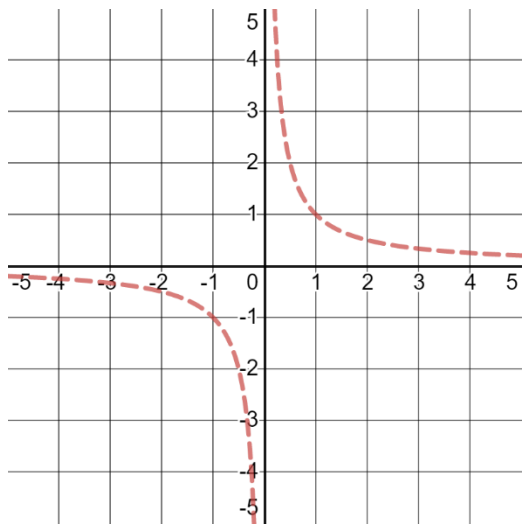
Practice: Sketch the image and write an equation for the transformations

$$\frac{1}{2}f(-(x + 1)) - 3$$

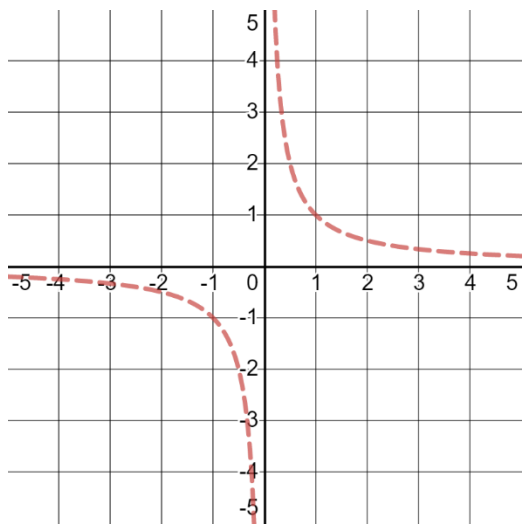


$$-f(-x + 1)$$

Practice: If $g(x) = \frac{1}{x}$, sketch the image function and write an equation for it using fractions instead of g

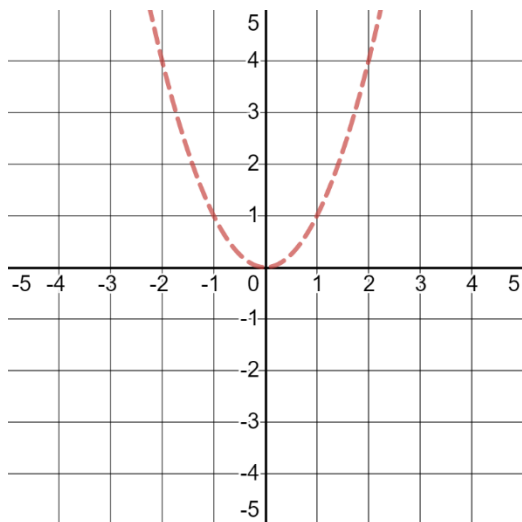


$$-2g(2x + 4) - 2$$



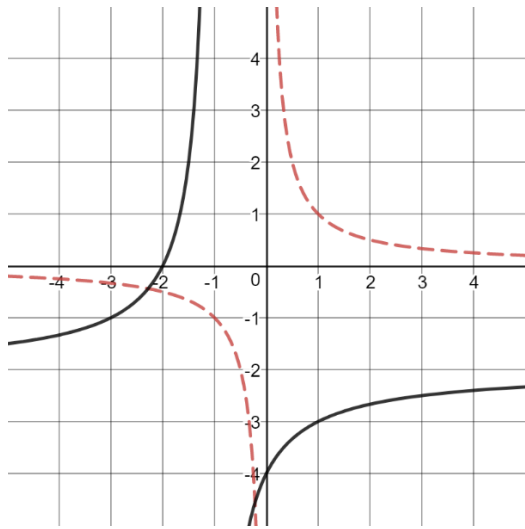
$$g\left(\frac{4-x}{2}\right) + 1$$

Practice: If $h(x) = x^2$, sketch the image function and write an equation for it using powers instead of h

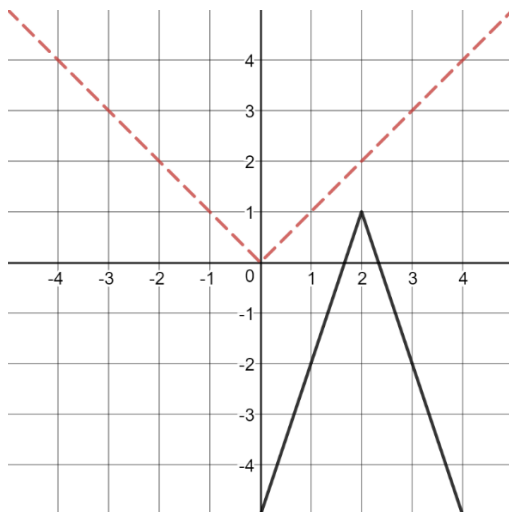
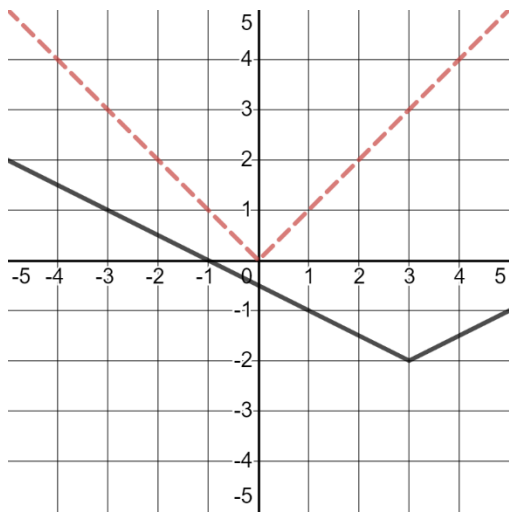


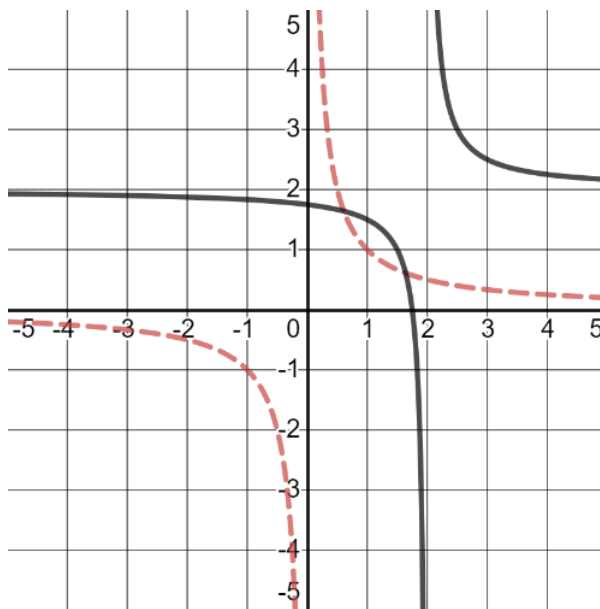
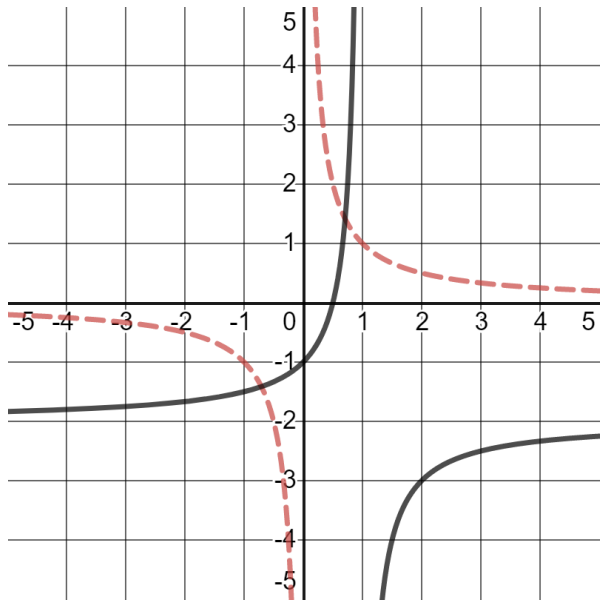
$$-h(4 - 2x) + 3$$

Example 3: To find the equation of a transformation we need to look at key characteristics of the function (zeros, y-intercept, asymptotes, reflections, etc)



Practice: Find the equation to the transformed graphs





Suggested Practice Problems: 1.3 page 39-43 #1, 2, 6, 7, 9-11, 13, 16-18
Textbook Reading: 1.3 page 32-37 Key Ideas on page 38
Next Class: Inverse functions as a transformation