Combining Stretches and Translations

Goal:

- Describe a complete transformation in the form $T(x) = a \cdot f\big(b(x-c)\big) + d$.
- Understands why the standard order is Stretch then Translate, and how changing the order can change the image function.
- Knows the shape of core function: x^2; $|x|$; $\frac{1}{x}$

Terminology:

- none

There is noting stopping us from doing a shift and stretch in tandem; however, we need to be mindful of the order.

When we say: "Perform a vertical expansion by a factor of 2, and then shift it up 2 units", we really mean

$(x, y) \mapsto (x, (2y)+2)$

But when we say: "Shift it up 2 units and then expand it vertically by a factor of 2", we are doing

$(x, y) \mapsto (x, 2(y+2))$

** **When combining transformations, the order we apply it is important!**

The order is determined by order of operations.

In function notation, the standard way of expressing a combination of transformations is:

left/right

$$\boxed{T(x) = a \cdot f\big(b(x-c)\big) + d}$$

up/down

a: vertical exp. $a>1$ expand $|a|<1$ compress

$a<0 \Rightarrow$ RoX

b: horiz. stretch $b>1$ compres., $|b|<1$ expand.

$b<0 \Rightarrow$ RoY

Which translates in mapping notation to:

$(x, y) \mapsto \left(\frac{1}{b}x + c, \; ay + d\right)$

$b(x-c) = A$

$x - c = \frac{1}{b}A$

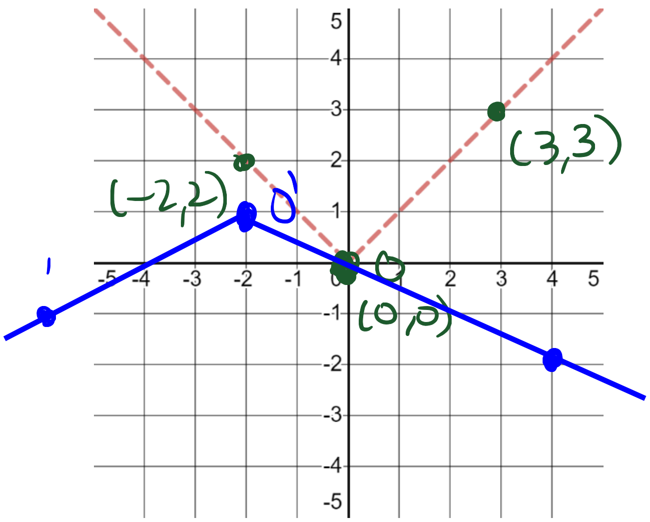
$x = \frac{1}{b}A + c$

stretch → shift

stretch then shift

$f(x+1) = |x+1|$

Example 1: Given that $f(x) = |x|$, sketch the image of the following and write an equation for the image that uses absolute value instead of f



$-f(0.5(x+2)) + 1$

$(x, y) \mapsto (2x-2, -y+1)$

$(0,0) \mapsto (-2, 1)$

$(3,3) \mapsto (4, -2)$

$(-2,2) \mapsto (-6, -1)$

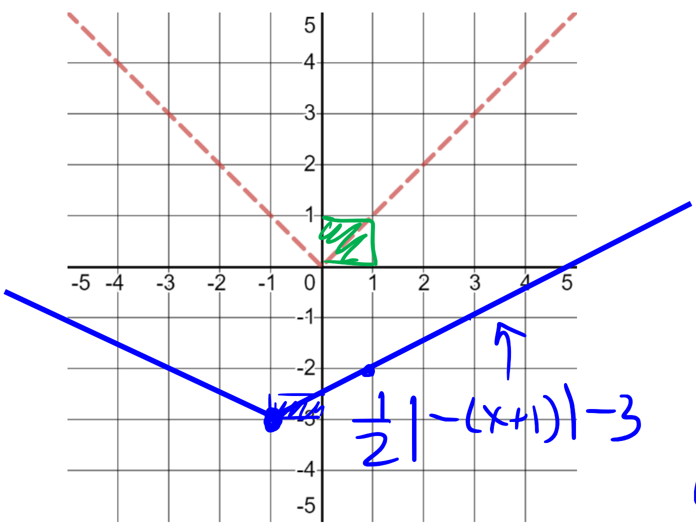
$0.5(x+2) = A$

$x+2 = 2A$

$x = 2A-2$

$g(x) = -|0.5(x+2)| + 1$

Practice: Sketch the image and write an equation for the transformations



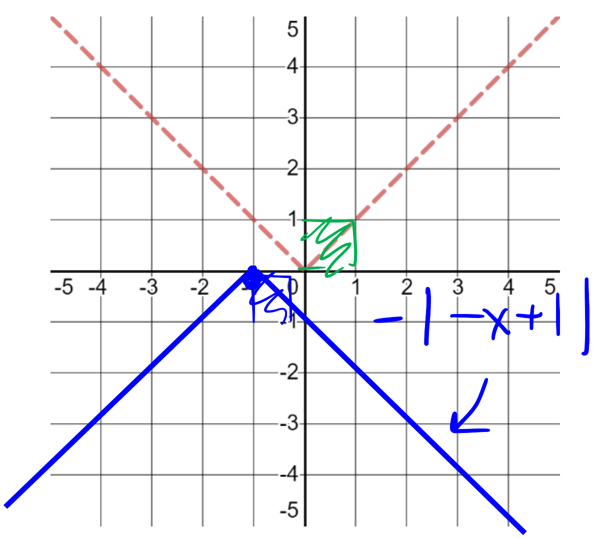
$\frac{1}{2}f(-(x+1)) - 3$

order: 1) add 1
2) times -1

map reverse order
1) divide -1
2) sub 1

$(x, y) \mapsto (-x - 1, \frac{1}{2}y - 3)$

Roy left vert comp. down



$-f(-x+1)$

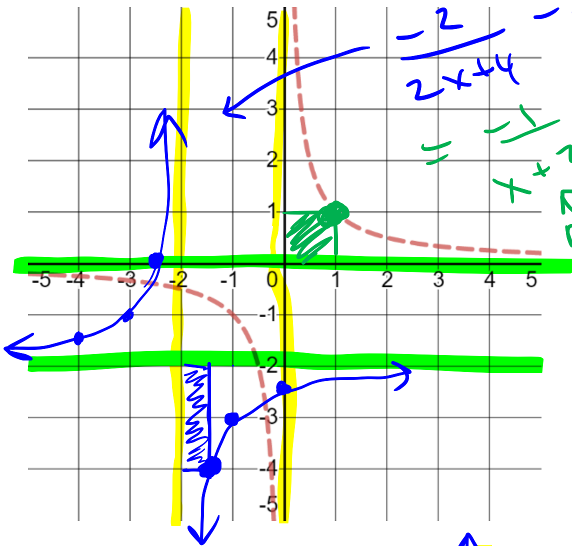
order: 1) times by -1
2) add by 1

Reverse for map
1) sub by 1
2) div. by -1

$(x, y) \mapsto (-(x-1), -y)$

Roy left Rox

Practice: If $g(x) = \frac{1}{x}$, sketch the image function and write an equation for it using fractions instead of g



$\frac{-2}{2x+4} - 2$

$= \frac{-1}{x+2} - 2$

No stretch

$$-2\underbrace{g(2x+4)}-2$$

Order: 1.) times by 2
2.) add by 4

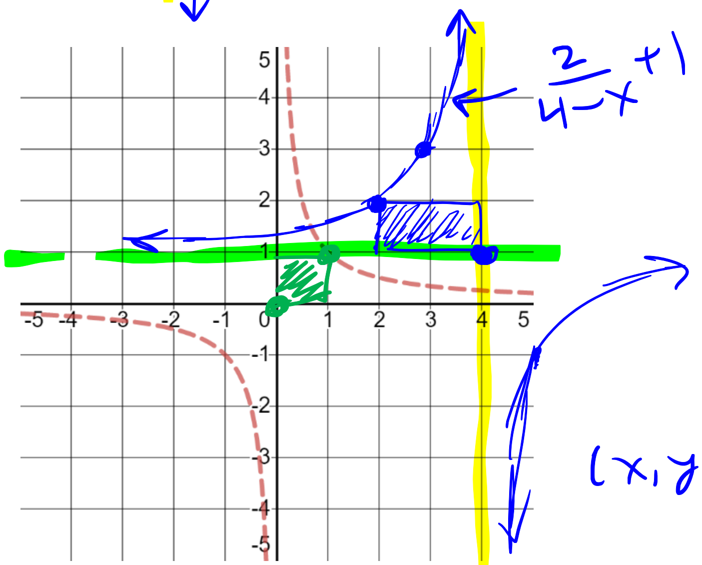
Reverse for map
1.) sub 4
2.) div. by 2

$(x,y) \longmapsto \left(\frac{x-4}{2}, -2y-2\right)$

$\frac{x}{2} - 2$

Horiz comp left

vert exp RoX down



$\frac{2}{4-x} + 1$

$$g\underbrace{\left(\frac{4-x}{2}\right)}_{\text{order}}+1$$

1.) times by -1
2.) add by 4
3.) divide by 2

map
1.) times by 2
2.) sub by 4
3.) div. by -1

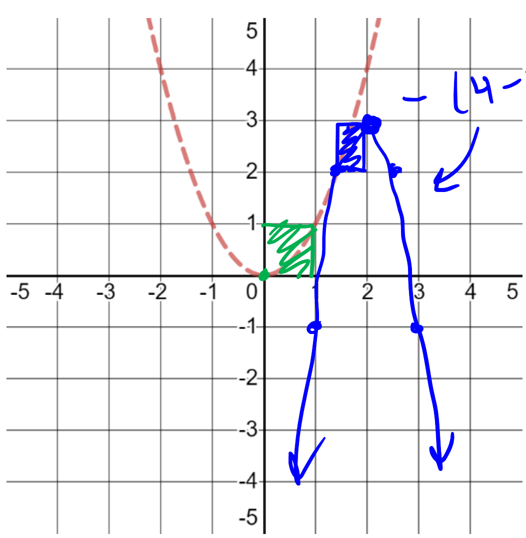
$(x,y) \longmapsto \left(-(2x-4), y+1\right)$

$-2x+4$

Horiz exp RoY Right

Up

Practice: If $h(x) = x^2$, sketch the image function and write an equation for it using powers instead of h



$-(4-2x)^2 + 3$

$$-h\underbrace{(4-2x)}_{\text{order}}+3$$

1.) times by -2
2.) add 4

map
1.) sub 4
2.) div. by -2

$(x,y) \longmapsto \left(\frac{x-4}{-2}, -y+3\right)$

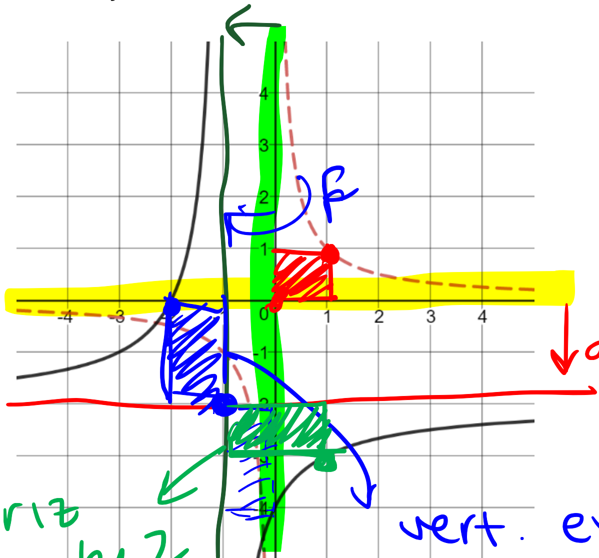
$-\frac{1}{2}x + 2$

RoX Up

Horiz. comp

RoY Right

Example 3: To find the equation of a transformation we need to look at key characteristics of the function (zeros, y-intercept, asymptotes, reflections, etc)



left 1

$g(x) = af(b(x+1)) - 2$

$g(x) = -2f(x+1) - 2$

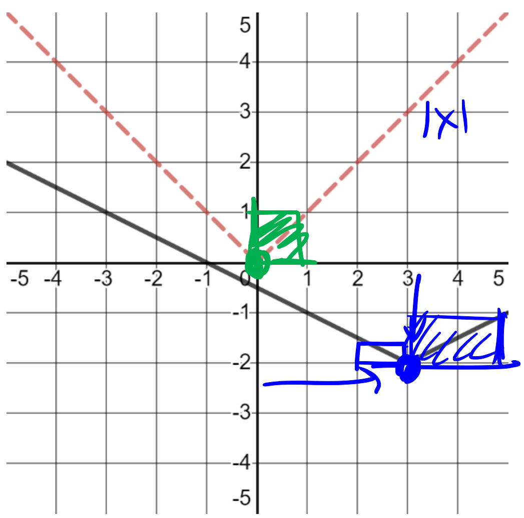
$= \dfrac{-2}{x+1} - 2$

vert. exp by 2 (R o x)

$f(\frac{1}{2}x) \overset{?}{=} 2f(x)$

$\frac{2}{x} = 2 \cdot \frac{1}{x}$

Practice: Find the equation to the transformed graphs



$c = +3$ (right)

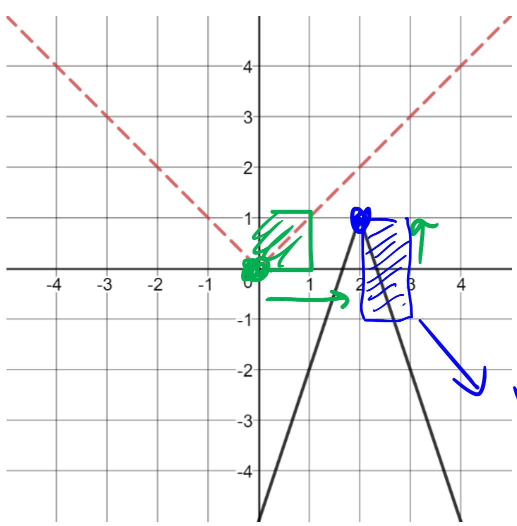
$d = -2$ (down)

horiz. exp by 2 $\Rightarrow b = \frac{1}{2}$

$f(b(x-c)) + d$

$f(\frac{1}{2}(x-3)) - 2$

$|\frac{1}{2}(x-3)| - 2 = \frac{1}{2}|x-3| - 2$



$c = 2$ (right)

$d = 1$ (up)

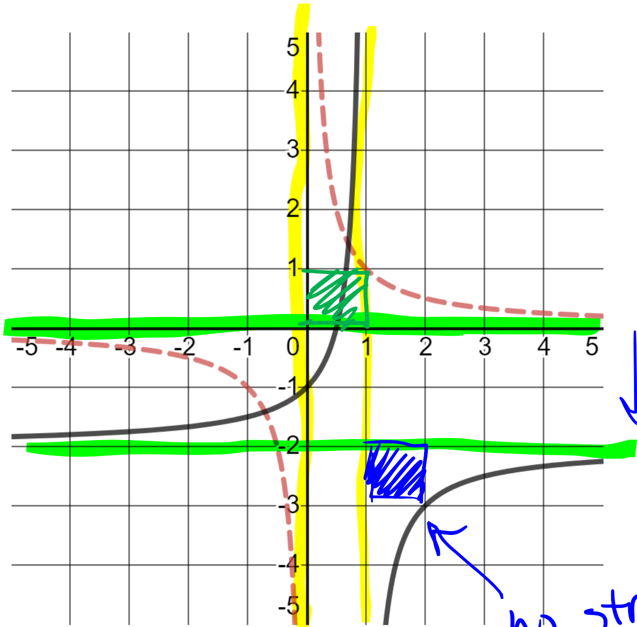
vertical exp by 2 and R o x $\Rightarrow a = -2$

$-2f(x-2) + 1 \Rightarrow y = -2|x-2| + 1$

→ right 1

$c = 1$

$d = -2$



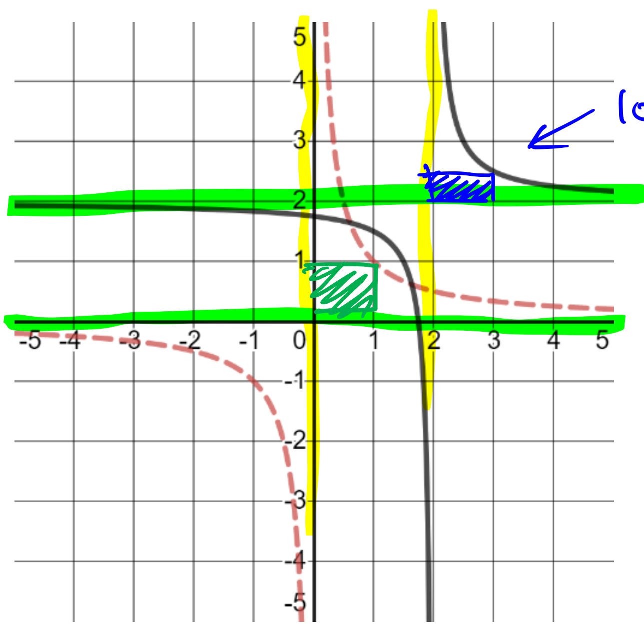
↓ down 2

$g(x) = -f(x-1) - 2$

$= -\left(\frac{1}{x-1}\right) - 2$

$= \frac{-1}{x-1} - 2$

no stretch, just Rox ⇒ $a = -1$



looks like vertical compres. by $\frac{1}{2}$ ⇒ $a = \frac{1}{2}$

↑ up 2 ⇒ $d = 2$

$g(x) = \frac{1}{2} f(x-2) + 2$

$= \frac{1}{2}\left(\frac{1}{x-2}\right) + 2$

$= \frac{1}{2x-4} + 2$

→ right 2 ⇒ $c = 2$

Suggested Practice Problems: 1.3 page 39-43 #1, 2, 6, 7, 9-11, 13, 16-18	
Textbook Reading: 1.3 page 32-37	Key Ideas on page 38
Next Class: Inverse functions as a transformation	