Combining Stretches and Translations

Goal:

- Describe a complete transformation in the form $T(x) = a \cdot f\big(b(x-c)\big) + d$.
- Understands why the standard order is Stretch then Translate, and how changing the order can change the image function.
- Knows the shape of core function: x^2; $|x|$; $\frac{1}{x}$

Terminology:

- none

There is noting stopping us from doing a shift and stretch in tandem; however, we need to be mindful of the order.

When we say: "Perform a vertical expansion by a factor of 2, and then shift it up 2 units", we really mean

$$(x, y) \mapsto (x, 2y+2)$$

But when we say: "Shift it up 2 units and then expand it vertically by a factor of 2", we are doing

$$(x, y) \mapsto (x, 2(y+2))$$

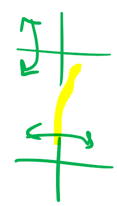
$(x,y) \mapsto (x, y+2) \mapsto (x, 2(y+2))$

** **When combining transformations, the order we apply it is important!**

on the map use order of operations.

In function notation, the standard way of expressing a combination of transformations is:

$$T(x) = a \cdot f\big(b(x-c)\big) + d$$



a: vertical stretch; $a>1$ expand; $|a|<1$ compress; $a<0$ RoX

b: horiz. stretch; $b>1$ compress; $|b|<1$ expand; $b<0$ RoY

Which translates in mapping notation to:

$$(x, y) \mapsto \left(\frac{1}{b}x + c,\ a \cdot y + d\right)$$

stretch $\to$ shift   stretch $\to$ shift

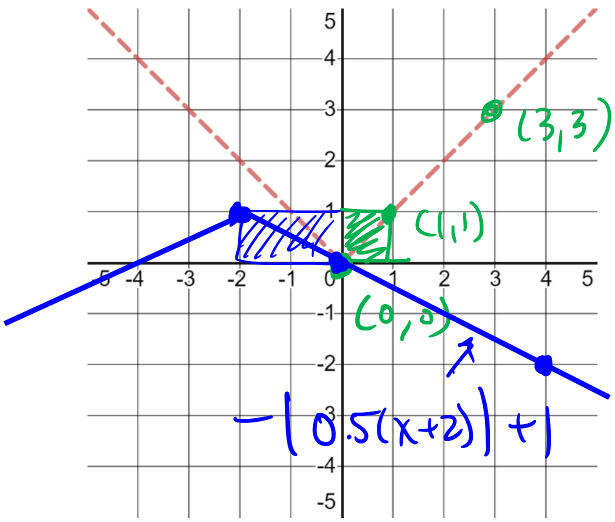
$b(x-c) = X$

$x - c = \frac{1}{b}X$

$x = \frac{1}{b}X + c$

Example 1: Given that $f(x) = |x|$, sketch the image of the following and write an equation for the image that uses absolute value instead of f

$$\left(-f(0.5(x+2))+1\right)$$ abstract



$(x,y) \mapsto (2x-2, -y+1)$

horiz. exp left   R ox   up

$0.5(x+2) = X \Rightarrow (x+2) = 2X$

$x = 2X - 2$

$(0,0) \mapsto (-2,1)$

$(3,3) \mapsto (4,-2)$

$(1,1) \mapsto (0,0)$

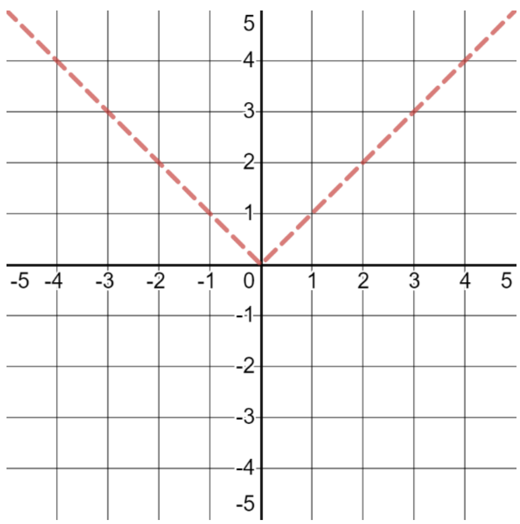
$f(x) = |x|$

$f(\heartsuit) = |\heartsuit|$

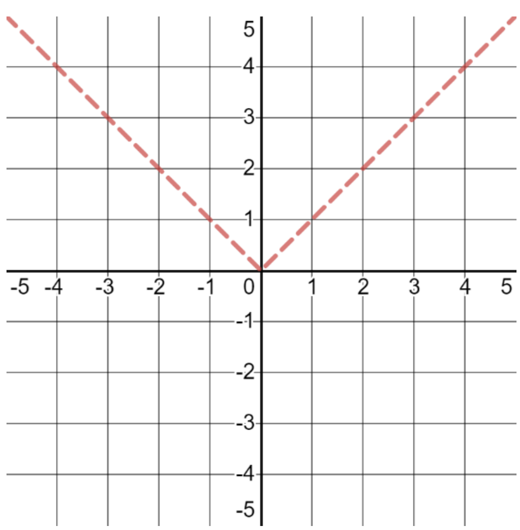
$f(-4) = |-4|$

$f(0.5(x+2)) = |0.5(x+2)|$

Practice: Sketch the image and write an equation for the transformations



$$\frac{1}{2}f\left(-(x+1)\right)-3$$



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$$-f(-x+1)$$

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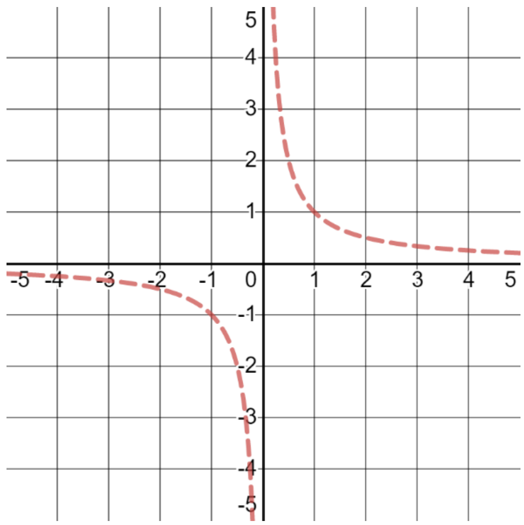
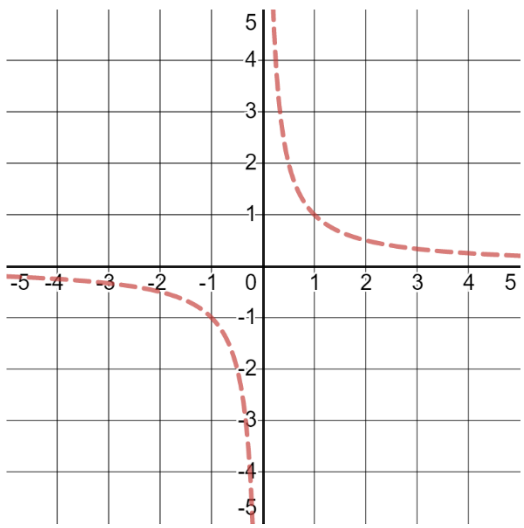
Practice: If $g(x) = \frac{1}{x}$, sketch the image function and write an equation for it using fractions instead of g



$$-2g(2x + 4) - 2$$



$$g\left(\frac{4 - x}{2}\right) + 1$$

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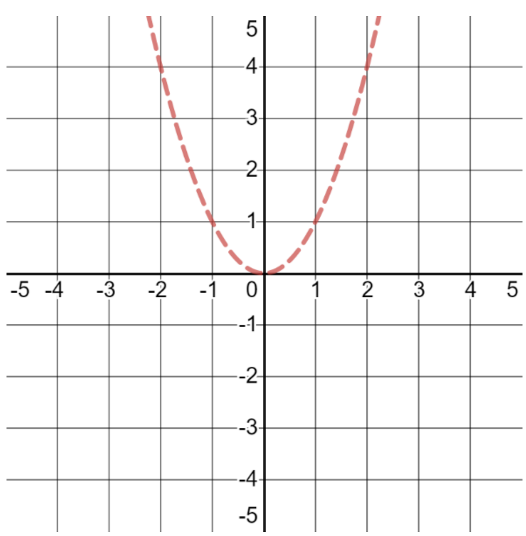
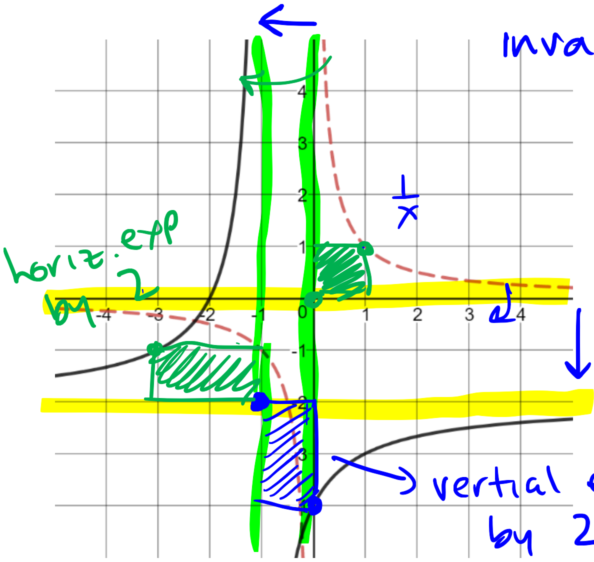
Practice: If $h(x) = x^2$, sketch the image function and write an equation for it using powers instead of h



$$-h(4 - 2x) + 3$$

Example 3: To find the equation of a transformation we need to look at key characteristics of the function (zeros, y-intercept, asymptotes, reflections, etc)



invariant points / lines

left 1 : $c=-1$

down 2 : $d=-2$

R o x : $a<0$

vert. exp. 2 : $a=-2$

$y=a \cdot f(b(x-c))+d$

$b<0$ R o y

horiz. exp by 2

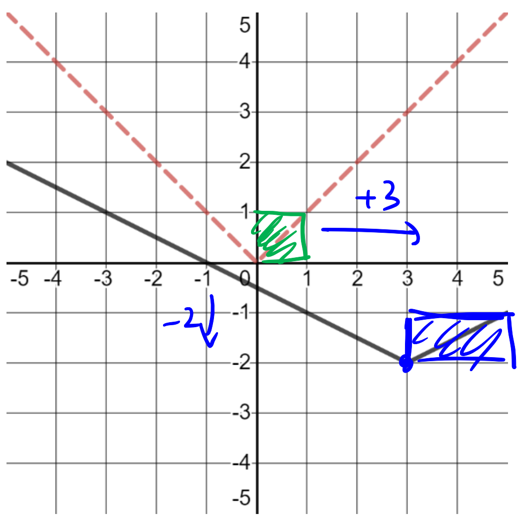
$b=-\frac{1}{2}$

$f(-\frac{1}{2}(x+1))-2$

$-2f(x+1)-2$

$\Rightarrow \boxed{-2\left(\frac{1}{x+1}\right)-2} \Leftrightarrow \frac{1}{-\frac{1}{2}(x+1)}-2$

Practice: Find the equation to the transformed graphs



$c=3$

$d=-2$

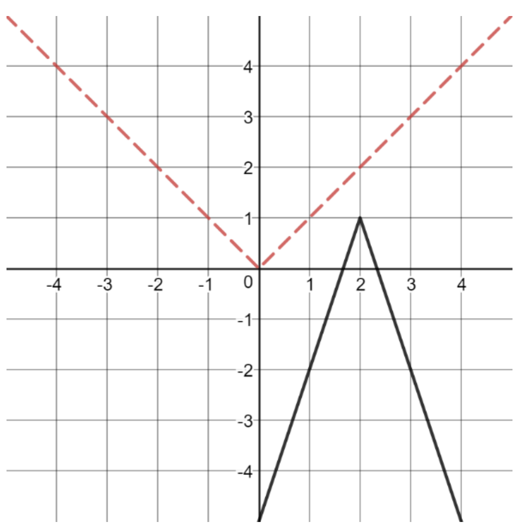
horiz. exp by 2

$b=\frac{1}{2}$

$af(b(x-c))+d$

$f(\frac{1}{2}(x-3))-2$

$|\frac{1}{2}(x-3)|-2=y$



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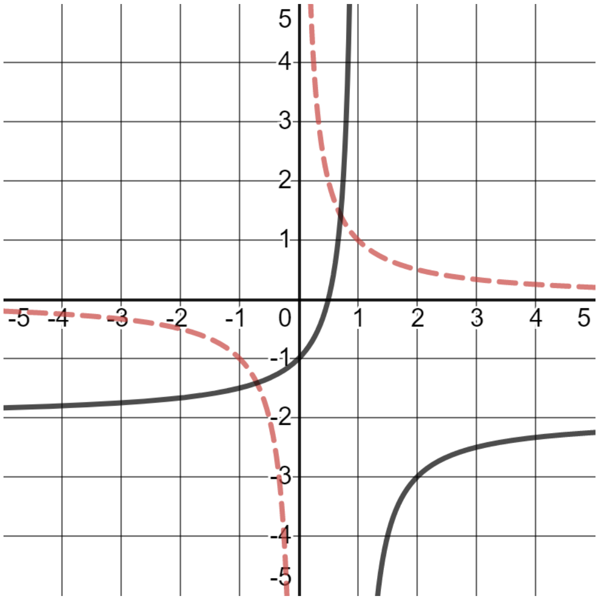

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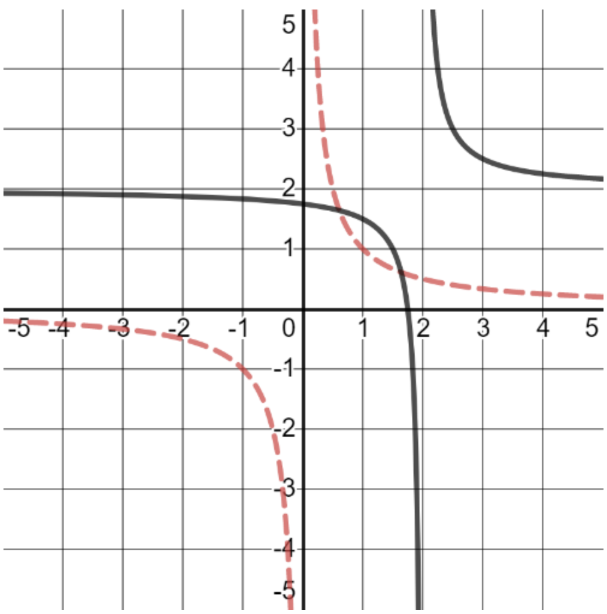

Suggested Practice Problems: 1.3 page 39-43 #1, 2, 6, 7, 9-11, 13, 16-18
Textbook Reading: 1.3 page 32-37 Key Ideas on page 38
Next Class: Inverse functions as a transformation