Transformations of Sine and Cosine

Goal:
- Can graph $a \cdot \sin(b(x-c)) + d$ based on transformations (or cosine).
- Can build the equation of a sinusoidal function based on its graph or characteristics.

Terminology:
- Phase Shift
- Vertical Displacement

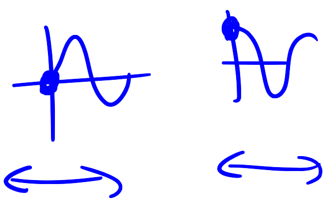
left/right up/down

We are going to graph functions of the form $a\sin(b(x-c)) + d$ just as we did with transformations.

Definition: The **phase shift** is the value of c

left/right movement

Characteristics effected are:



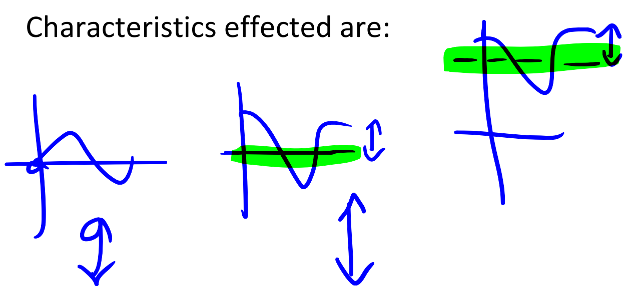
where we start
(sine usually in the middle)
(cosine usually at the top)

**Note that when we talk about phase shift, the transformed function is in standard form with b factored out

Definition: The **vertical displacement** is the value of d

up/down movement

Characteristics effected are:



where my midline occurs

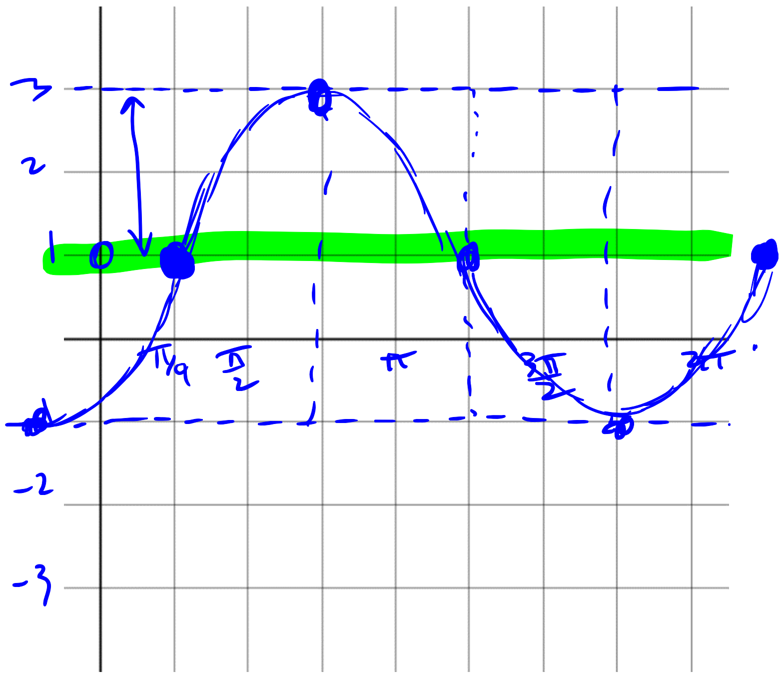
normally @ $y=0$, after we are @ $y=d$

Example: Graph $f(\theta) = 2\sin\left(\theta - \frac{\pi}{4}\right) + 1$

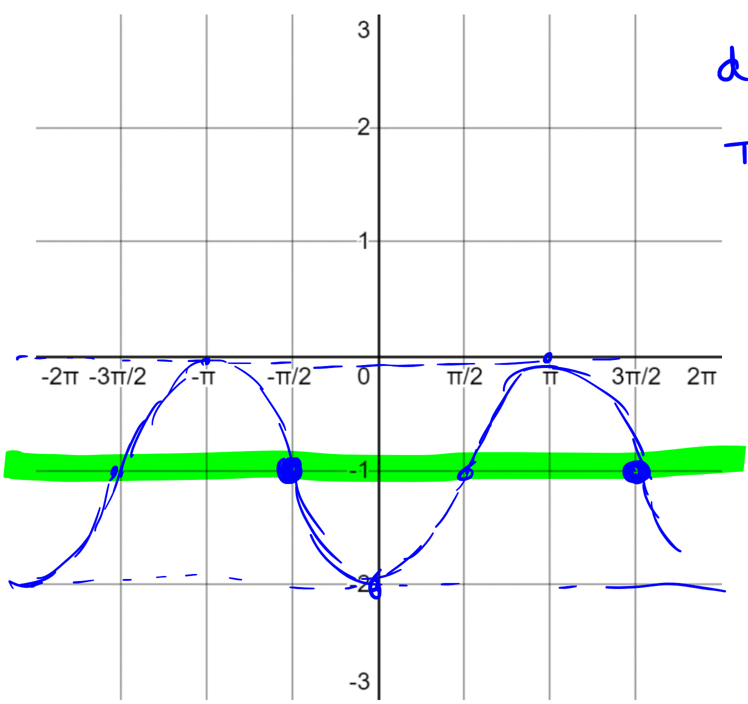
- Identify the midline from the vertical displacement
- Use the amplitude to find the max and min lines
- Use the phase shift to identify the starting point
- Split the period into quarters.

right $\pi/4$

$T = 2\pi$



Practice: Graph $g(\theta) = -\sin\left(\theta + \frac{\pi}{2}\right) - 1$

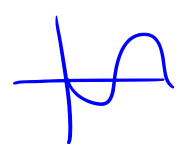


$a = 1$ (amplitude)

$d = -1$ (midline)

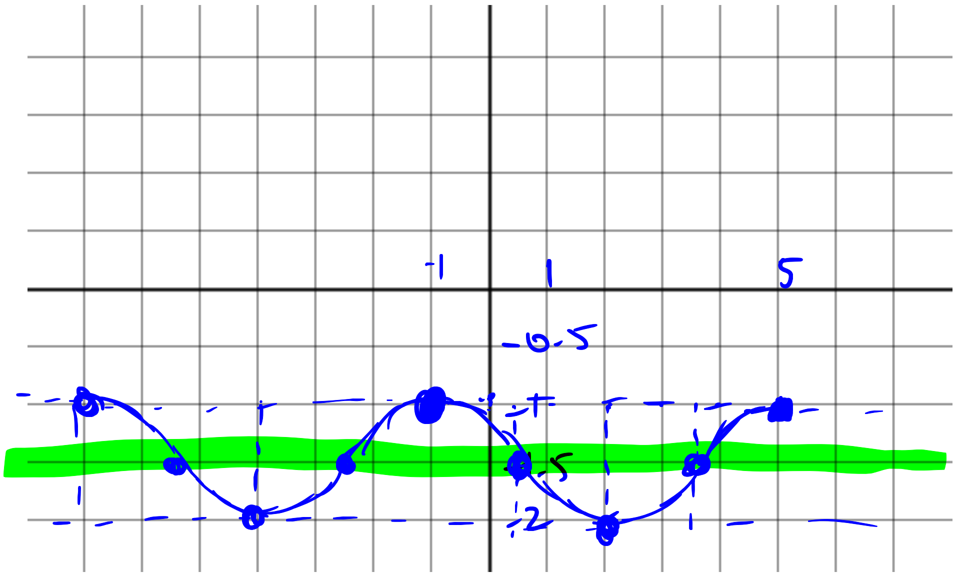
$\pi/2$ left

$T = 2\pi$



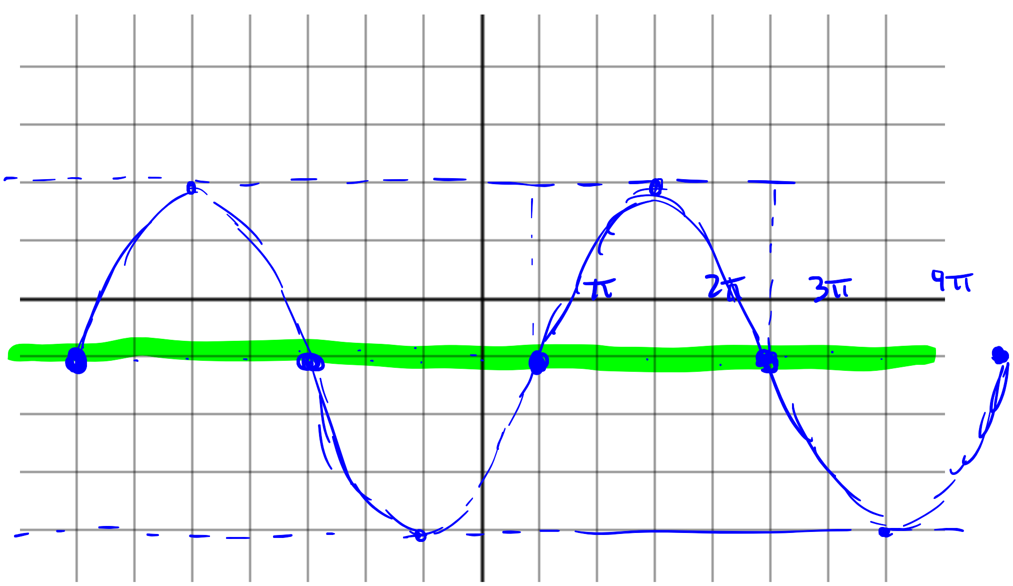
left

Practice: Graph $h(\theta) = 0.5\cos\left(\frac{\pi}{3}(\theta + 1)\right) - 1.5$



$T = \frac{2\pi}{\pi/3} = 6$

Practice: Graph $k(\theta) = 3\sin\left(\frac{1}{2}\left(\theta - \frac{\pi}{2}\right)\right) - 1$



amp = 3

mid line = -1

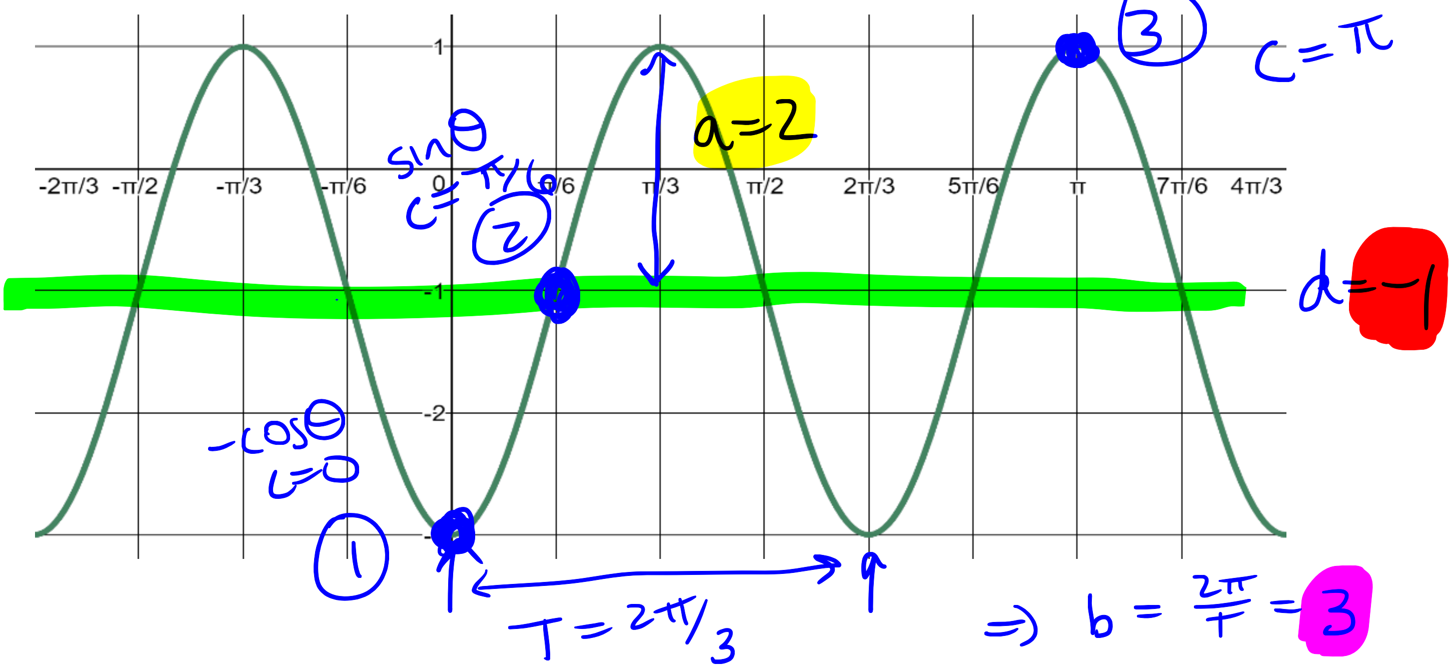
$T = \frac{2\pi}{1/2} = 4\pi$

$\pi/2$ right

When trying to determine the equation of a sinusoidal function, do the same steps

- Identify the midline
- Use the midline to determine the amplitude
- Use the distance between peaks to find the period
- Decide if you want a cosine or sine equation. Pick the place to start and identify the phase shift.

Example: Determine 3 different equations that could describe the following function.



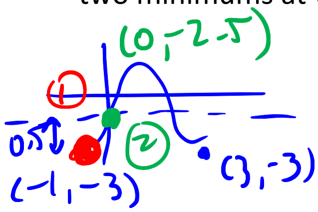
$\Rightarrow b = \frac{2\pi}{T} = 3$

① $-2\cos 3(\theta) - 1$

③ $2\cos 3(\theta - \pi) - 1$

② $2\sin 3(\theta - \pi/6) - 1$

Example: Determine two equations (one sine, one cosine) that could describe a sinusoidal function that has two minimums at $(-1, -3)$ and $(3, -3)$ and has an amplitude of 0.5.



$a = 0.5$

$y = -2.5 = d$

$b = \frac{2\pi}{4} = \frac{\pi}{2}$

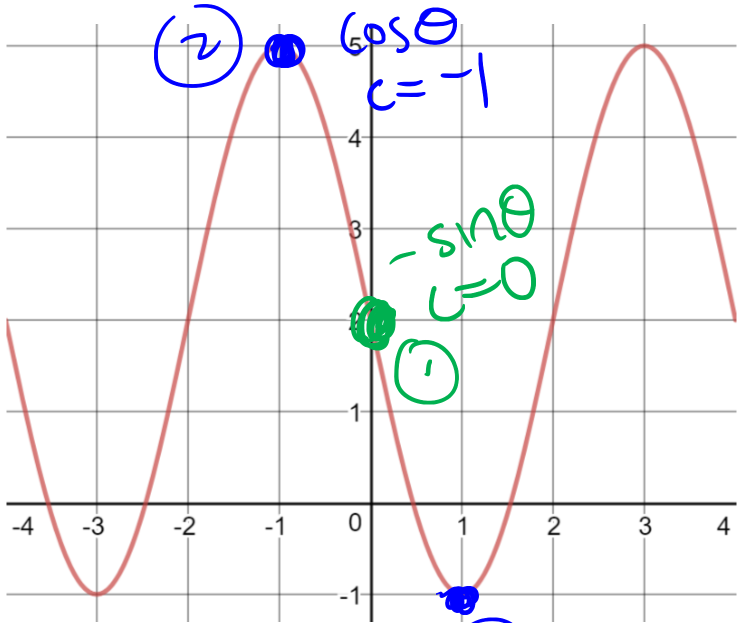
@ ① $-\cos\theta$, $c = -1$

$\Rightarrow y = -0.5\cos\frac{\pi}{2}(\theta + 1) - 2.5$

@ ② $\sin\theta$, $c = 0$

$\Rightarrow y = 0.5\sin\frac{\pi}{2}\theta - 2.5$

Practice: Determine 3 different equations that could describe the following function



(2) $\cos\theta$, $c = -1$

$-\sin\theta$, $c = 0$ (1)

(3) $-\cos\theta$, $c = 1$

$d = 2 \quad a = 3$

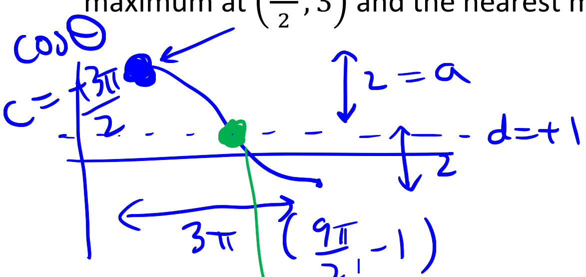
$T = 4 \Rightarrow b = \frac{\pi}{2}$

(1) $y = -3\sin\frac{\pi}{2}\theta + 2$

(2) $y = 3\cos\frac{\pi}{2}(\theta + 1) + 2$

(3) $= -3\cos\frac{\pi}{2}(\theta - 1) + 2$

Practice: Determine two equations (one sine, one cosine) that could describe a sinusoidal function that has a maximum at $\left(\frac{3\pi}{2}, 3\right)$ and the nearest minimum is at $\left(\frac{9\pi}{2}, -1\right)$.



$-\sin\theta$

$C = \frac{3\pi}{2} + \frac{3\pi}{2}$

$= 3\pi$

$T = 6\pi$

$b = \frac{1}{3}$

(1) $2\cos\frac{1}{3}\left(x - \frac{3\pi}{2}\right) + 1$

(2) $-2\sin\frac{1}{3}(x - 3\pi) + 1$

Suggested Practice Problems: 5.2 # 1-2 (radians), 4-9, 12-16, 18, 20, 22-24, 27, 28	
Textbook Reading: Reading: Textbook page 238-248 Key Ideas page 249	
Next Class: Modelling Trig Equations	