Transformations of Sine and Cosine

Goal:

- Can graph $a \cdot \sin(b(x-c)) + d$ based on transformations (or cosine).
- Can build the equation of a sinusoidal function based on its graph or characteristics.

Terminology:

- Phase Shift
- Vertical Displacement

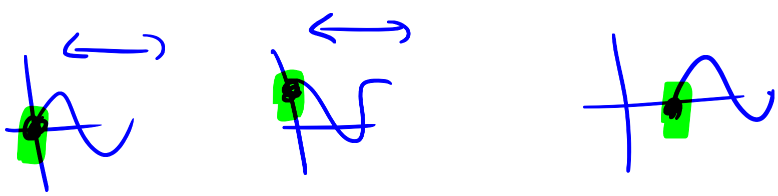
horiz. vert

We are going to graph functions of the form $a\sin(b(x-c)) + d$ just as we did with transformations.

Definition: The **phase shift** is the value of c

left/right shift

Characteristics effected are:



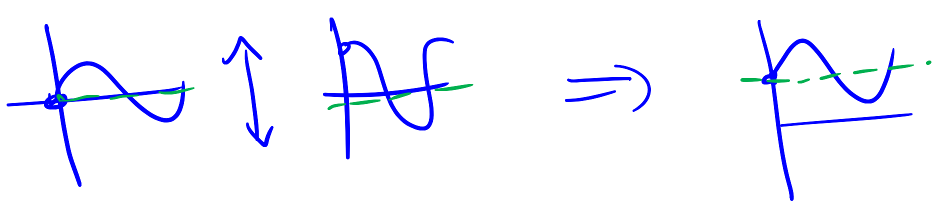
changes start position

**Note that when we talk about phase shift, the transformed function is in standard form with b factored out

Definition: The **vertical displacement** is the value of d

up/down shift

Characteristics effected are:



change the position of the midline.

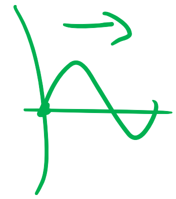
amp.

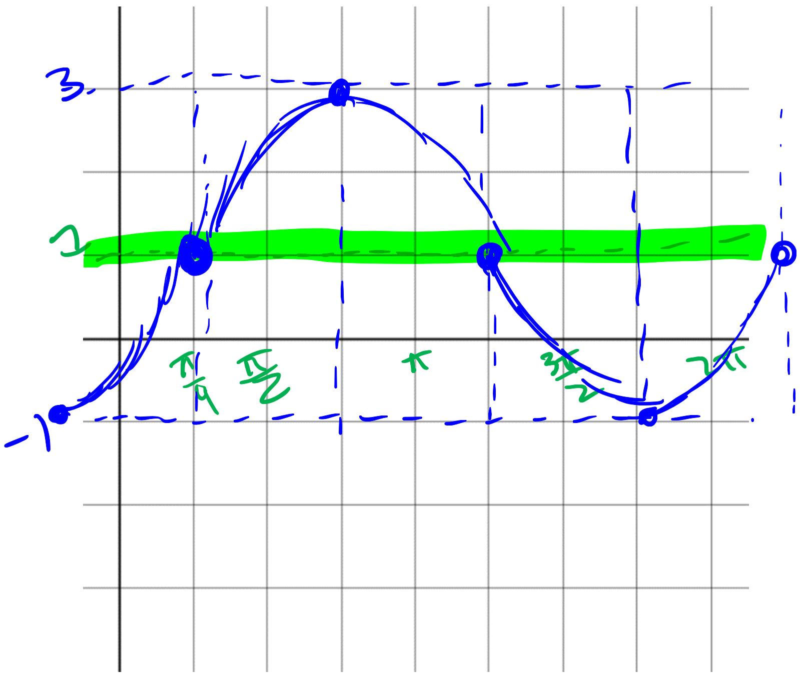
Example: Graph $f(\theta) = 2\sin\left(\theta - \frac{\pi}{4}\right) + 1$ midline

- Identify the midline from the vertical displacement
- Use the amplitude to find the max and min lines
- Use the phase shift to identify the starting point
- Split the period into quarters.

right $\pi/4$

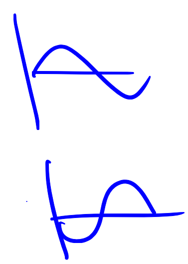
$T = 2\pi$

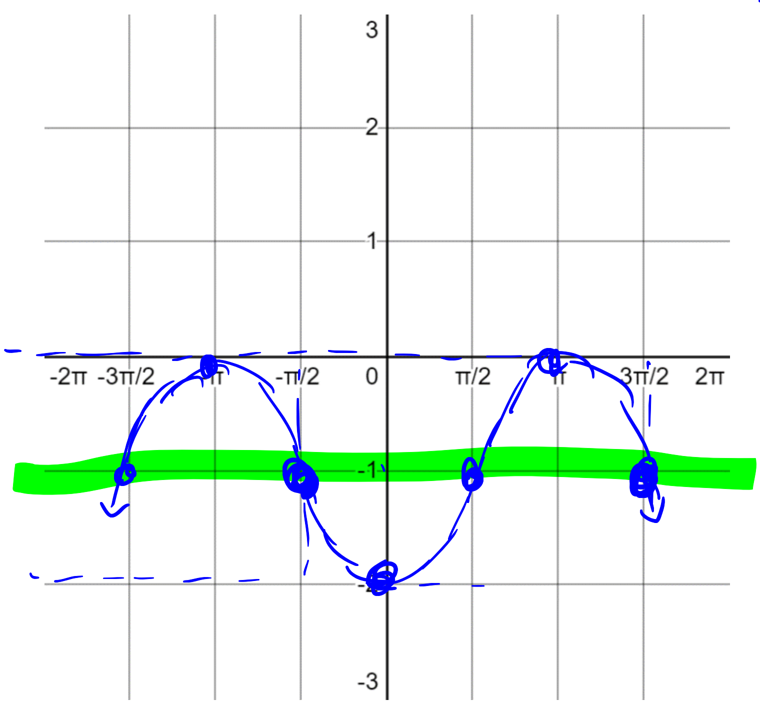




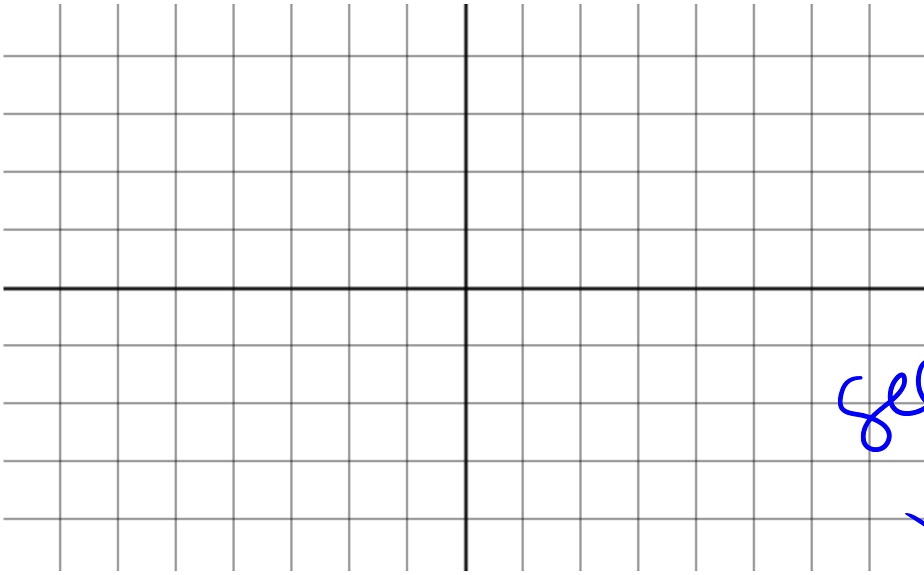
Practice: Graph $g(\theta) = -\sin\left(\theta + \frac{\pi}{2}\right) - 1$

$T = 2\pi$





Practice: Graph $h(\theta) = 0.5\cos\left(\frac{\pi}{3}(\theta + 1)\right) - 1.5$



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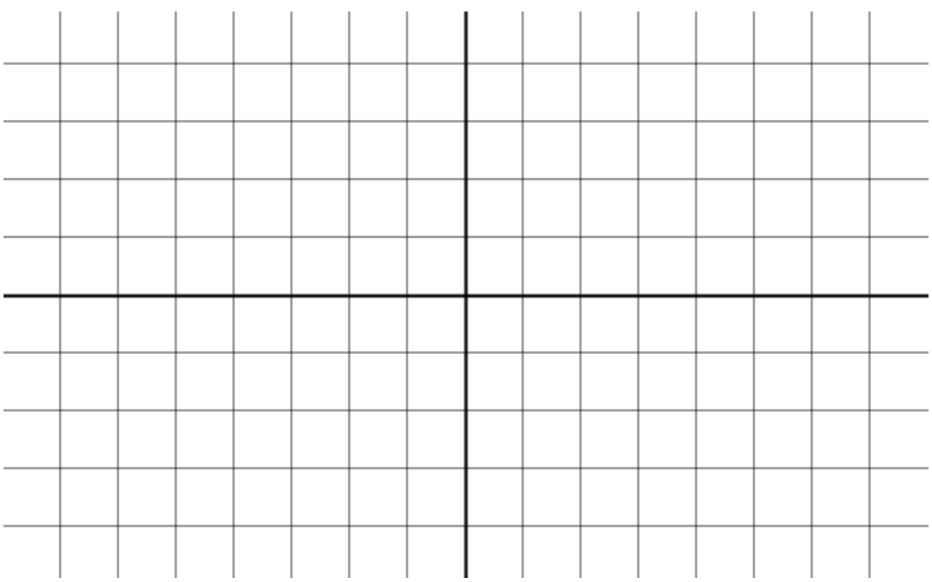
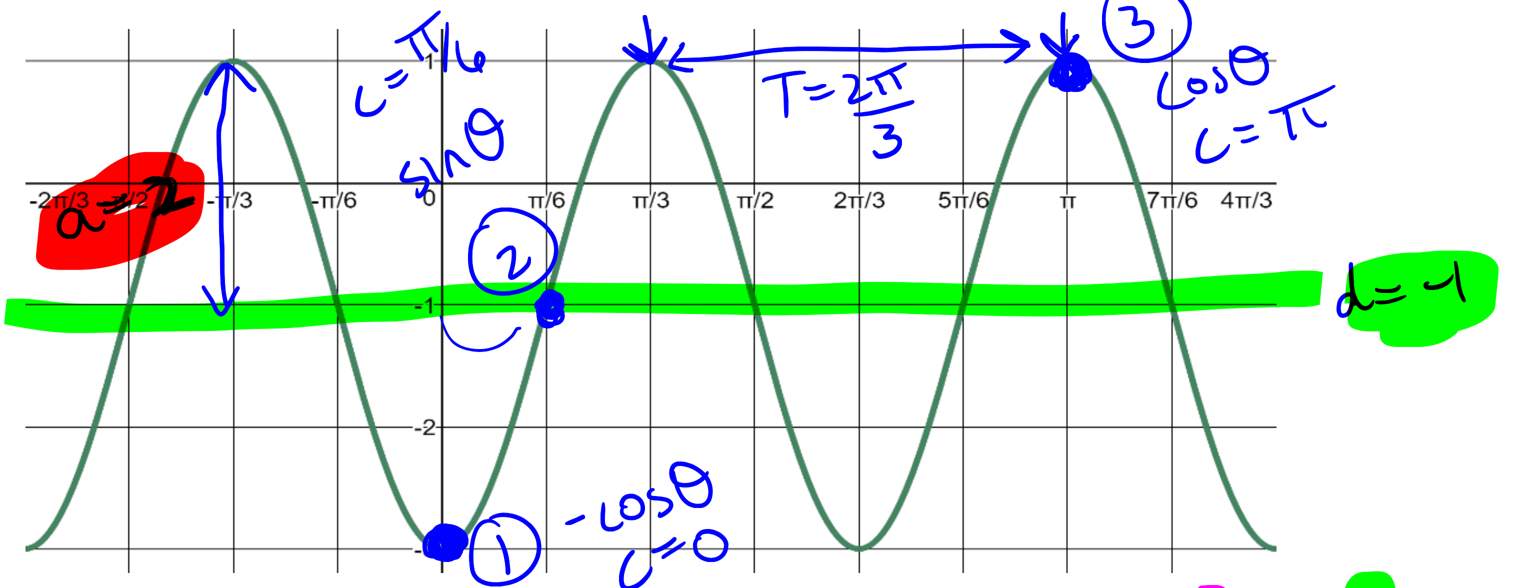
Practice: Graph $k(\theta) = 3\sin\left(\frac{1}{2}\left(\theta - \frac{\pi}{2}\right)\right) - 1$



When trying to determine the equation of a sinusoidal function, do the same steps

- Identify the midline
- Use the midline to determine the amplitude
- Use the distance between peaks to find the period
- Decide if you want a cosine or sine equation. Pick the place to start and identify the phase shift.

Example: Determine 3 different equations that could describe the following function



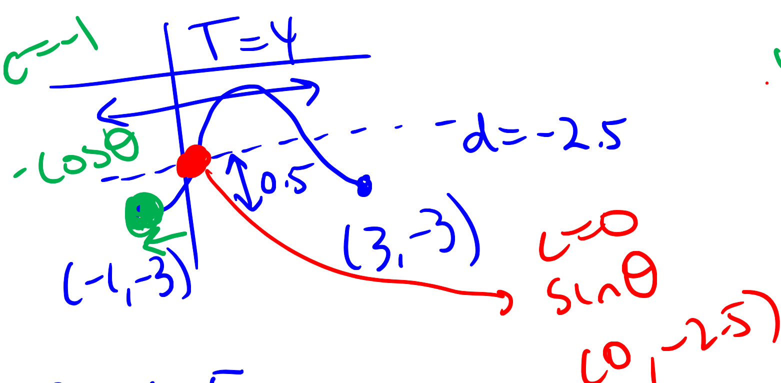
$b = \frac{2\pi}{T} = \frac{2\pi}{\frac{2\pi}{3}} = 3$

① $y = -2\cos 3\theta - 1$

② $y = 2\sin 3\left(\theta - \frac{\pi}{6}\right) - 1$

③ $y = 2\cos 3(\theta - \pi) - 1$

Example: Determine two equations (one sine, one cosine) that could describe a sinusoidal function that has two minimums at $(-1, -3)$ and $(3, -3)$ and has an amplitude of 0.5.



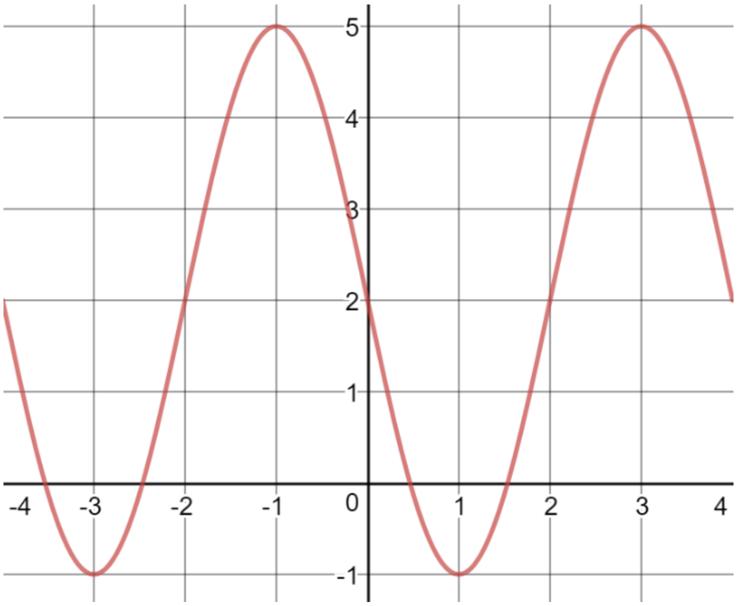
$y = -0.5\cos\frac{\pi}{2}(\theta + 1) - 2.5$

$y = 0.5\sin\frac{\pi}{2}\theta - 2.5$

$a = 0.5$

$T = 4 \Rightarrow b = \frac{2\pi}{4} = \frac{\pi}{2}$

Practice: Determine 3 different equations that could describe the following function



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Practice: Determine two equations (one sine, one cosine) that could describe a sinusoidal function that has a maximum at $\left(\frac{3\pi}{2}, 3\right)$ and the nearest minimum is at $\left(\frac{9\pi}{2}, -1\right)$.

Suggested Practice Problems: 5.2 # 1-2 (radians), 4-9, 12-16, 18, 20, 22-24, 27, 28	
Textbook Reading: Reading: Textbook page 238-248 Key Ideas page 249	
Next Class: Modelling Trig Equations	