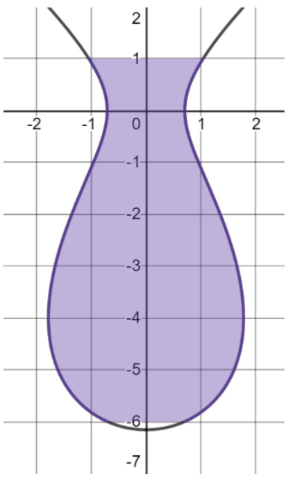
Volumes Part 1: Cross Sections and Rotations

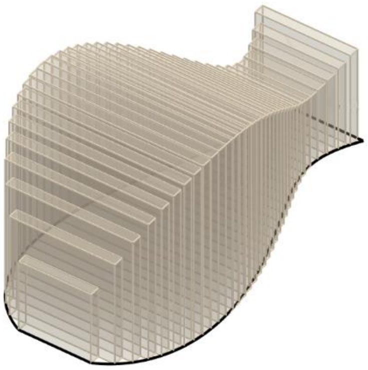
Goal: • Can determine the volume of solids using geometric slices • Can derive the volume equations to pyramids and spheroids • Can determine the volume of solids after rotation around the x or y axis
Terminology: • None

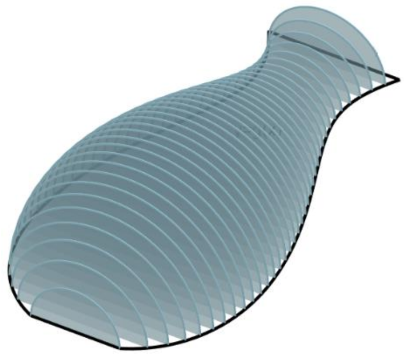
Determine the area of the shaded region given the curve is

$$(2x)^2 = \frac{y^3}{3} + 2y^2 + 2$$



Consider a few differnet ways of making a vase out of this region



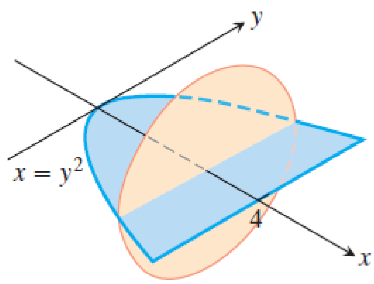


To find the volume of a region, we just need to add up the cross sectional areas:

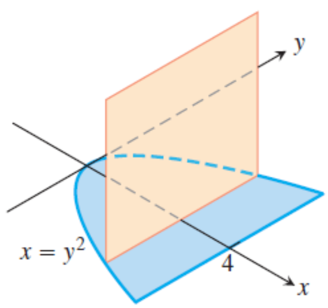
$$V = \lim_{n\to\infty} \sum_{k=1}^{n} A(x_k)\,\Delta x = \int_a^b A(x)\,dx$$

The solid lies between the planes perpendicular to the x-axis at $x = 0$ and $x = 4$. The cross sections perpendicular to the x-axis between these planes run between $x = y^2$.

a. The cross sections are circular discs with diameters in the xy-plane



b. The cross sections are squares with bases in the xy-plane



c. The cross sections are squares with **diagonals** in the xy-plane

d. The cross sections are equilateral triangles with bases in the xy-plane

d. The cross sections are parabolas with bases in the xy-plane (the height and base of the parabolas are the same)

A fun consequence is that now we can derive equations for volumes of known geometric objects

Example: Show that the volume of a cylinder and cone with radius r and height h is

$$V_1 = \pi r^2 h, \qquad V_2 = \frac{1}{3}\pi r^2 h$$

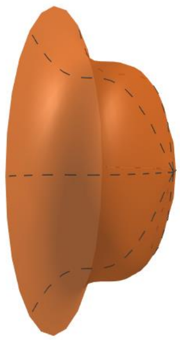
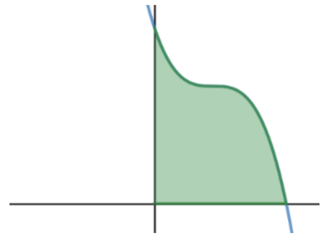
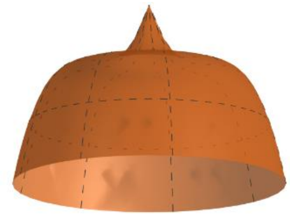
Practice: Show that the volume of a prism with constant cross secrtional area A and height h is just

$$V = Ah$$

Practice: Show that the volume of a square based pyramid with base side length a and height h is

$$V = \frac{1}{3}a^2 h$$

We are going to finish by briefly talking about how we can generate circular cross sections and that is by simply revolving a region around an axis to generate a volume. Consider the region bound between $x = 0$, $y = 0$ and the curve $y = f(x)$ as shown below

Practice: Find the volume of the solid generated by roating the area between the x and y-axis and the curve $y = 1 - x^2$ if you were to rotate it about the x or y axis.

Practice: Consider the volume formed when the function $f(x) = \frac{1}{x}$ where $x \in [1, b]$ is rotated over the x-axis. What is the volume? What happens when $b \to \infty$?

Practice Problems: 7.3 # 1-20, 27-38	
Textbook Readings: 7.3 page 383-385	
Workbook Practice: page 328-337	
Next Class: Volumes using washers and shells	