Graphing f from f' and f''

Goal:
- Can use the first derivative to identify when the function increases and decreases
- Can use the second derivative to identify when the function is concave up and concave down
- Can graph a function to have correct extrema and concavity

Terminology:
- None

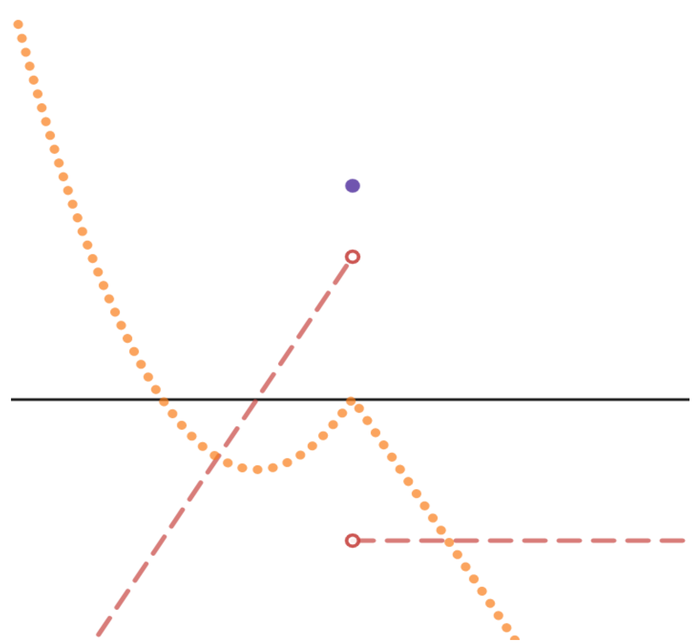
Recall our definitions for the continuous function $f: [a, b] \to \mathbb{R}$
(change inequality direction for decreasing, minimums, and concave down).

Increasing:
- **(First Principles)** If $\forall\, x, y \in [a, b]$ with $x < y$ we get $f(x) < f(y)$, then f is increasing on $[a, b]$
- **(Calculus)** If $f'(x) > 0\ \forall\, x \in (a, b)$ then f is increasing on $[a, b]$

Maximum:
- **(First Principles)** If $f(c) \geq f(x)\ \forall\, x \in [a, b]$, then $x = c$ is an absolute maximum
- **(First Principles)** If $f(c) \geq f(x)$ for every x on some open interval around c, then $x = c$ is a local maximum
- **(Calculus)** If $f'(x) > 0$ when $x \in (a, c)$ and $f'(x) < 0$ when $x \in (c, b)$ then $x = c$ is a local maximum
- **(Calculus)** If $f''(x) < 0$ when $x \in (a, b)$ and $f'(c) = 0$ for $c \in (a, b)$ then $x = c$ is a local maximum

Concavity:
- **(First Principles)** If $f'(x)$ is increasing on $[a, b]$ then it is concave up on $[a, b]$
- **(Calculus)** If $f''(x) > 0\ \forall\, x \in (a, b)$ then f is concave up on $[a, b]$ (assuming $f'(a)$ and $f'(b)$ are definied)

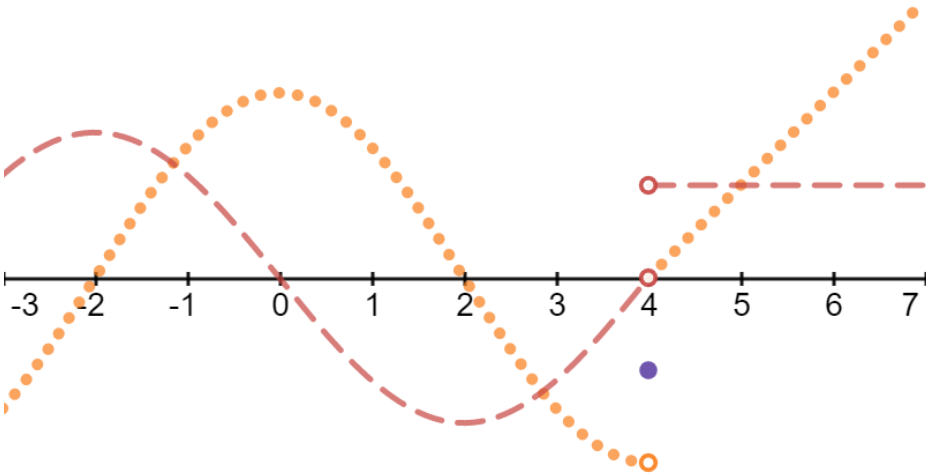
Inflection Points:
- **(Calculus)** If $f''(x)$ is positive or negative when $x \in (a, c)$ and $f''(x)$ is negative or positive when $x \in (c, b)$ then $x = c$ is an inflection point. (f'' needs to change sign)



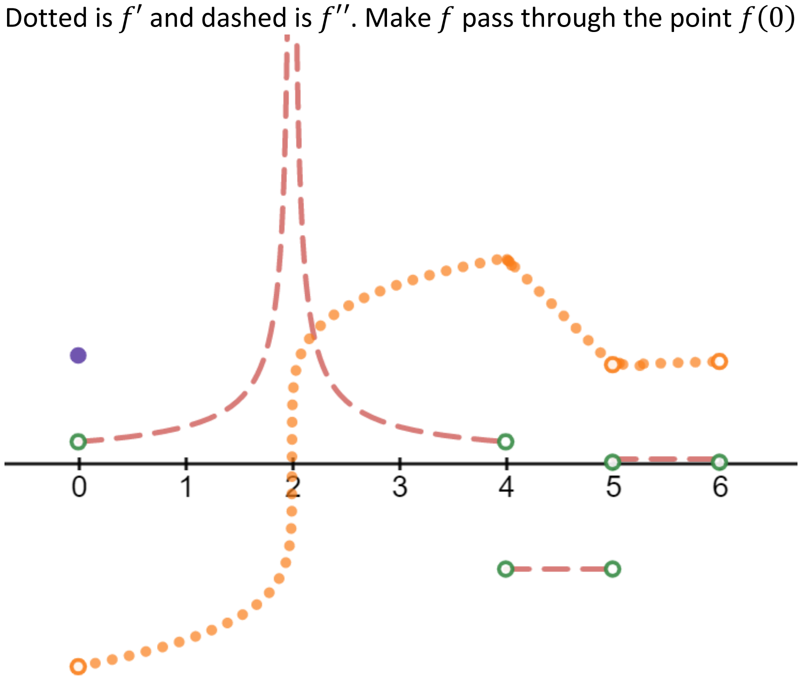
Practice:

- Sketch the continuous functions f
- label the max and mins and inflection points
- state the intervals f is increasing and intervals it is concave up.

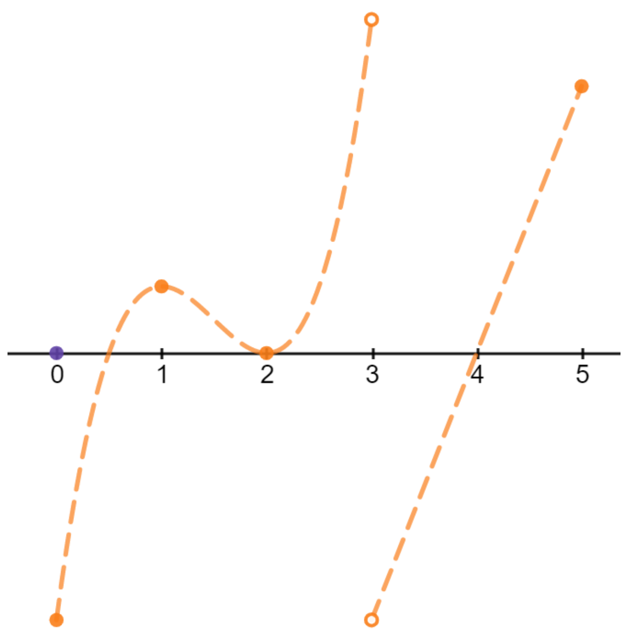
1. Dotted is f' and dashed is f''. Make f pass through the point $f(4) = -1$ shown below. The domain of f is $\mathbb{R}$



2. Dotted is f' and dashed is f''. Make f pass through the point $f(0) = 1$ shown below. The domain of f is $[0, 6]$



3. Dotted curve is f'. Make f pass through the point $f(0) = 0$ shown below. The domain of f is $[0, 5]$



4. The curve has the following properties.

$f(-2) = 6$

$f(0) = 5$

$f(2) = -1$

$f'(-2) = 0$

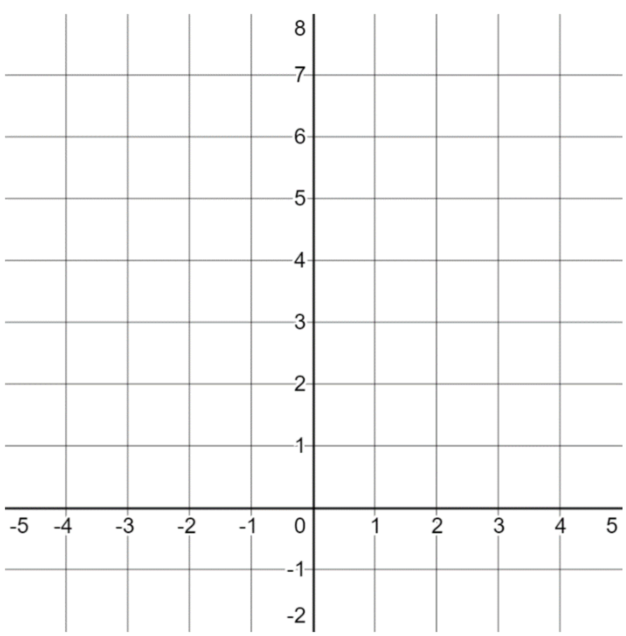
$f'(x) > 0$ when $|x| > 2$

$f'(x) < 0$ when $|x| < 2$

$f'(2)$ is undefined

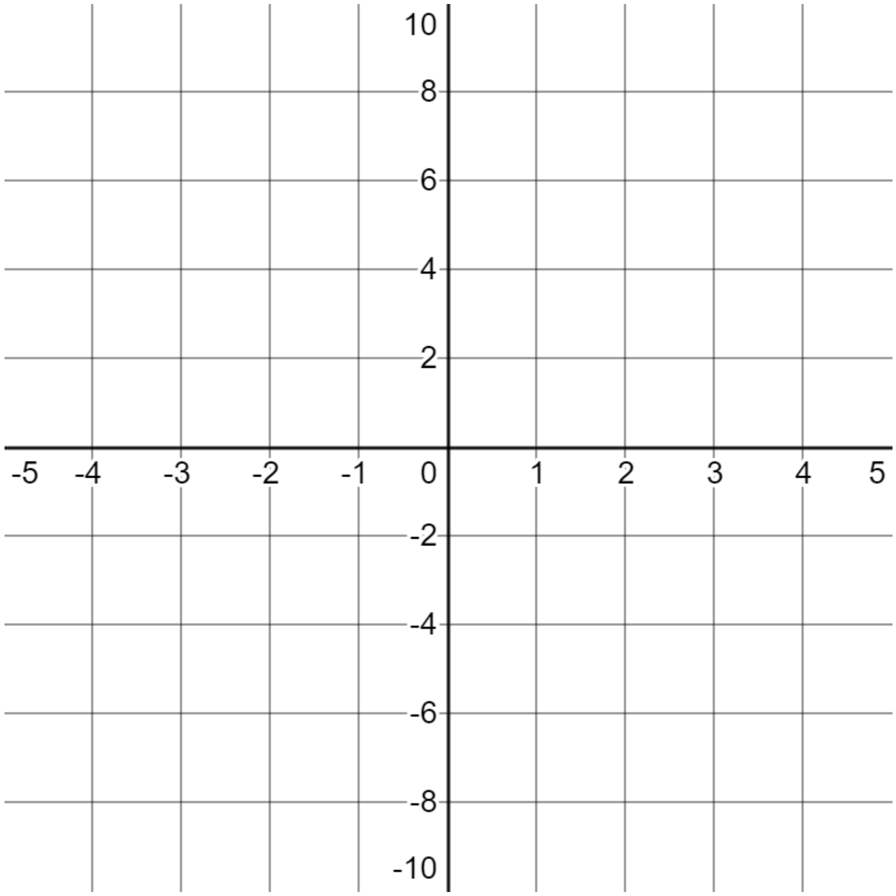
$f''(x) > 0$ when $x > 2$

$f''(x) < 0$ when $x < 2$



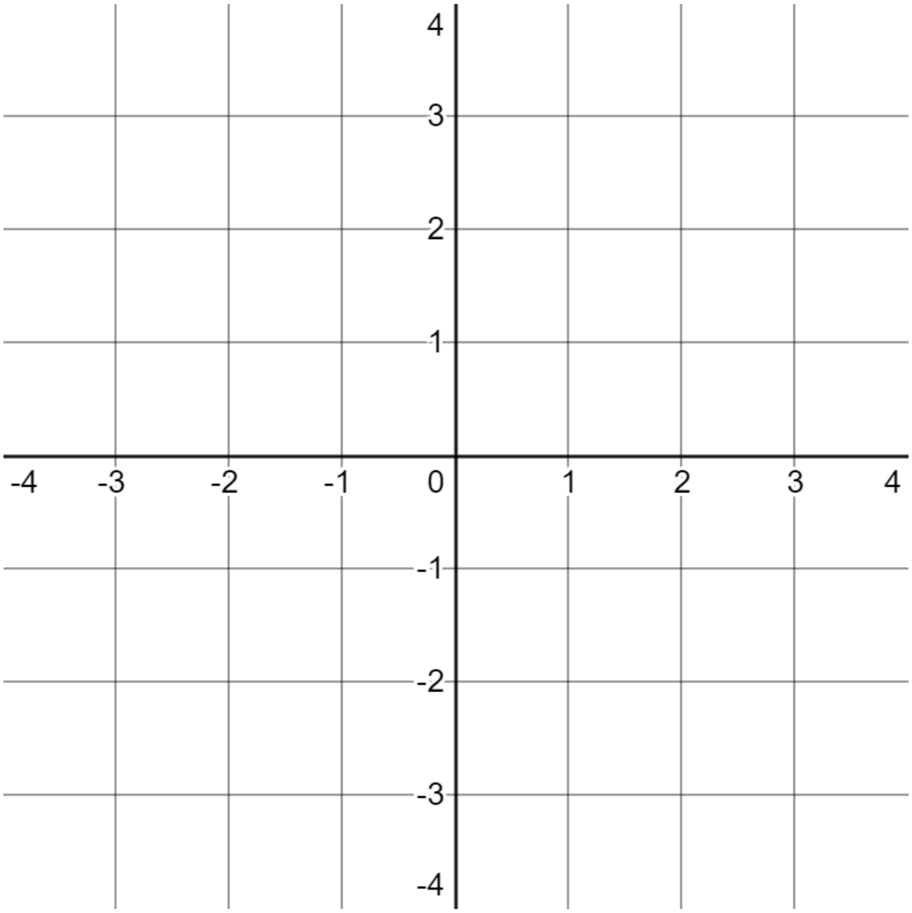
5.

$$f(x) = -\frac{1}{5}x^5 - \frac{3}{4}x^4 + 5$$



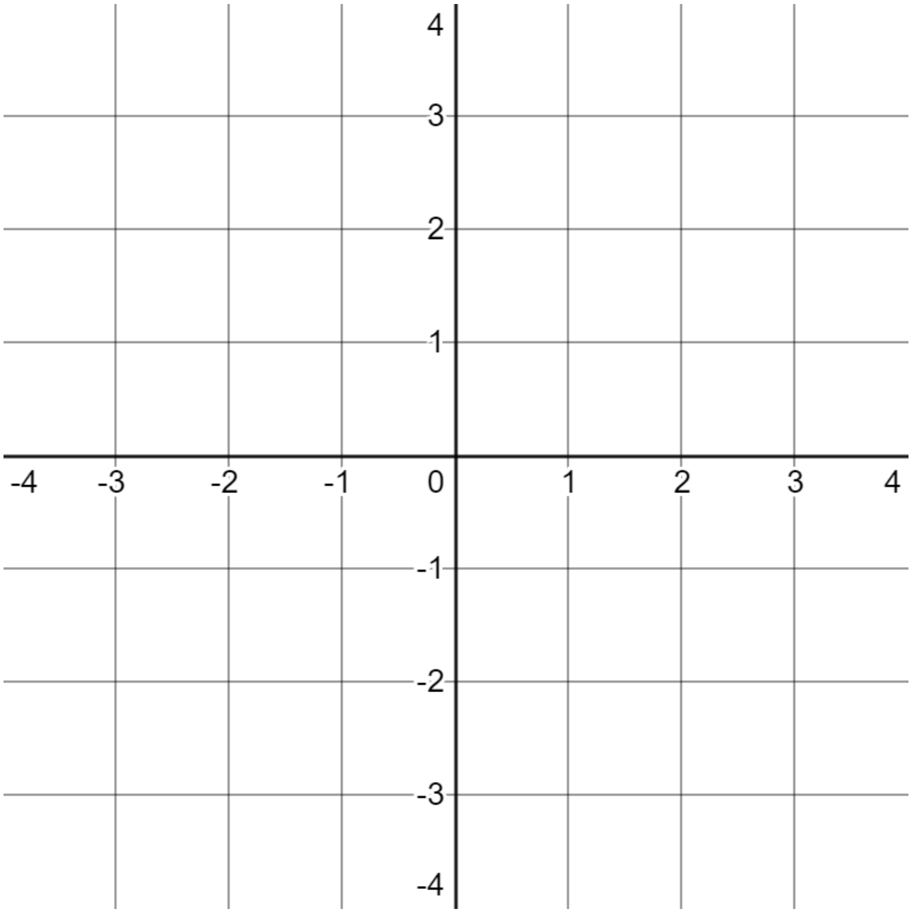
6.

$$f(x) = 4e^{-\frac{1}{x^2}}$$



7.

$$f(x) = \arctan x^2$$



Practice Problems: 4.3: # 31-36, 45-48
Textbook Readings: 4.3 page 201-202
Workbook Practice: page 194-197, 200-203
Next Day: Optimization