# Log Laws

**Goal:**

- Can derive the product law using exponent laws
- Can derive the power law and quotient law from the product law.
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**Terminology:**

- Product Law
- Quotient Law
- Power Law
- Change of Base Law

In grade 9 and 10 you learned about the exponent laws and know that

$$b^n \cdot b^m = b^{n+m} \qquad (b^n)^m = b^{nm} \qquad b^{-1} = \frac{1}{b}$$

$$\underbrace{b \cdot b \cdots b}_{n} \cdot \underbrace{b \cdot b \cdots b}_{m} = \underbrace{b \cdot b \cdots b}_{n+m} \qquad \underbrace{b^n \cdot b^n \cdots b^n}_{m} = b^{\overbrace{n+\cdots+n}^{m}} \qquad \text{By definition}$$

Using function notation if $g(x) = b^x$ then the above laws for exponents give defining characteristics

$$g(n) \cdot g(m) = g(n+m) \qquad g(n)^m = g(nm) \qquad g(-x) = \frac{1}{g(x)}$$

Logarithms, being the inverse of exponentials have similar laws:

**Product Law**: $\log_b(m \cdot n) = \log_b m + \log_b n$

**Power Law**: $\log_b x^n = n \cdot \log_b x$

**Quotient Law**: $\log_b \left(\frac{m}{n}\right) = \log_b m - \log_b n$

**Product Law Proof**:

**Power Law Proof:**

**Quotient Law Proof:**

**Practice**: Use log laws to simplify the following as a single log:

$$\log_2 7 - 2\log_2 3 + \log_2 6$$

$$3\ln 6 - \ln 9 - \ln 8$$

$$2\log(12+3) - (\log 5 + \log 4)$$

$$\log e - \ln 10$$

We need to be careful about the domain when we simplify log functions:

**Example**: Simplify the following and state the overall domain.

$$f(x) = -\log_6 \sqrt{x} + 4.5 \log_6 x$$

**Practice**: Simplify the following and state the overall domain

$$g(x) = \log x + 2\log(x+1) - \log\big((x+1)(x-2)\big)$$

Aside from simplifying, expanding can help us simply an expression if we can evaluate logs of the smaller pieces using:

**Inverse Property**: $\log_b b^x = x$ and $b^{\log_b x} = x$

**Practice**: Given that $\log_2 5 = A$ and $\log_2 3 = B$ simplify the following.

$$\log_2 40\sqrt{5}$$

$$\log_2 \left(\frac{36}{5}\right)^3$$

**Change of Base Law**: $\log_b a = \frac{\log_x a}{\log_x b} =$

**Proof**:

**Practice**: Transform the following functions to the common log and the natural log.

$f(x) = \log_2 x$

$g(x) = -2\log_{100} x$

$h(x) = \log_{e^3}(x-1) + 2$

$k(x) = \log_{10^n} x$

Finally, a word about how **log scales** alter perspective. When things grow logarithmically (very slowly) we can change our base to whatever we want (common or natural). In this way, we will get some function

$$f(x) \propto \log x$$

And if $\Delta f = f(x_2) - f(x_1) = n$, then that means $n \propto \log x_2 - \log x_1 = \log\left(\frac{x_2}{x_1}\right)$ so $\frac{x_2}{x_1} \propto 10^n$

**Example**: There are a lot of ways to measure earthquake sizes, but the most common measure is the Moment Magnitude which describes the amount of energy released according to the formula:

$$M = \frac{2}{3}\log E + 1.22$$

Where $E$ is the energy released as kg of TNT.

a. The 2011 earthquake in Japan had a moment magnitude of 9.1. Compare that to the earthquake in Vancouver in 2015 which had a magnitude of 4.8. How much stronger was the earthquake in Japan?

b. If the magnitude increases by 1 how much will the energy increase by?

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| **Suggested Practice Problems**: 8.3 page 400-403 # 1-3, 5, 6, 8-13, 17-20, C1 |
| **Textbook Reading:** 8.3 page 392-399<br>Key Ideas on page 400 |
| **Next Class:** Practice solving and modelling log and exponential equations |

Exponential Growth

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