Solving Trig Equations

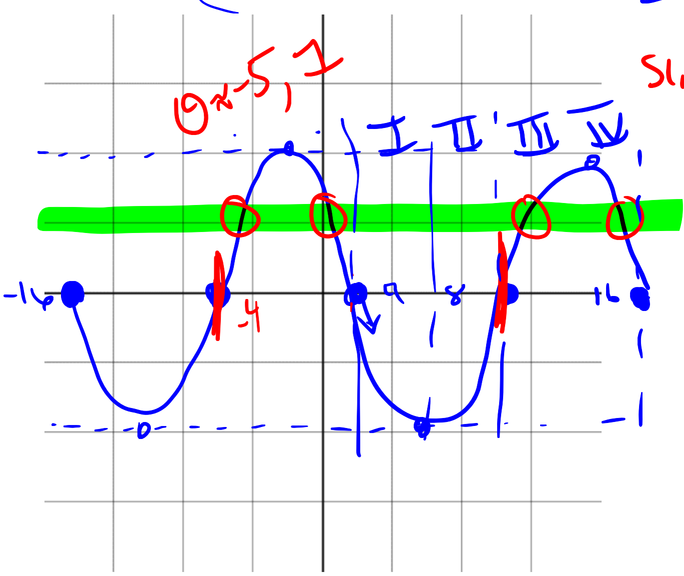
Goal:
• Can solve trig equations graphically and algebraically.
Terminology:
• None

In addition to using algebra to solve trig equations, we can graph the trig equations and look for approximate (or accurate solutions using technology) solutions in the intersections.

Example: Determine the general solution to the following equation:

$$\frac{-2\sin\left(\frac{\pi}{8}(\theta - 2)\right)}{-2} = \frac{1}{-2}$$

$T = \frac{2\pi}{\pi/8} = 16$



$\sin^{-1}\left(\sin\left(\frac{\pi}{8}(\theta-2)\right) = -\frac{1}{2}\right)$

$\Rightarrow \frac{\pi}{8}(\theta-2) = -\frac{\pi}{6}, -\frac{5\pi}{6} + 2\pi n$

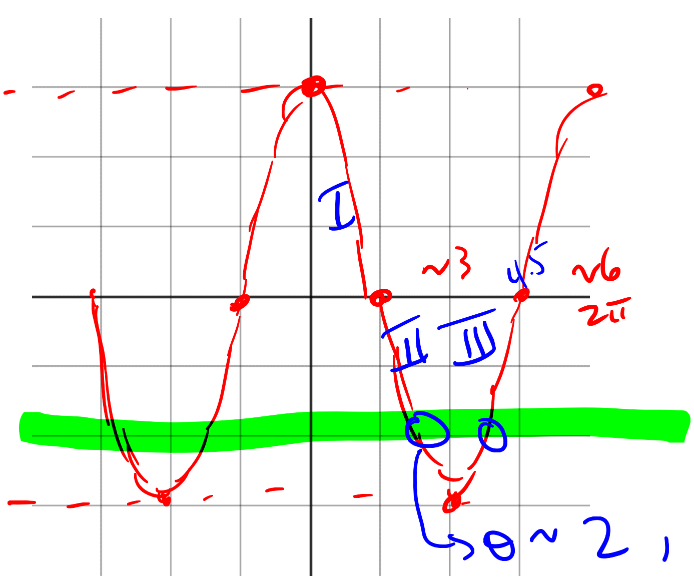
$\theta - 2 = -\frac{8}{6}, -\frac{8}{6}\cdot 5 + 16n$

$\boxed{\theta = \frac{2}{3}, -\frac{14}{3} + 16n, n \in \mathbb{Z}}$

$0.66, -4.66$

Practice: Find the general solution to the following

$$3\cos\theta = -2$$



$\cos\theta = -\frac{2}{3}$

$\theta = \arccos(-2/3)$

$= 2.3, \pi + 0.84$

$3.98, + 2\pi n$

$\pi - 2.3 = 0.84$

$\theta_1 = 2.3 + 2\pi n$

$\theta_2 = 3.98 + 2\pi n$

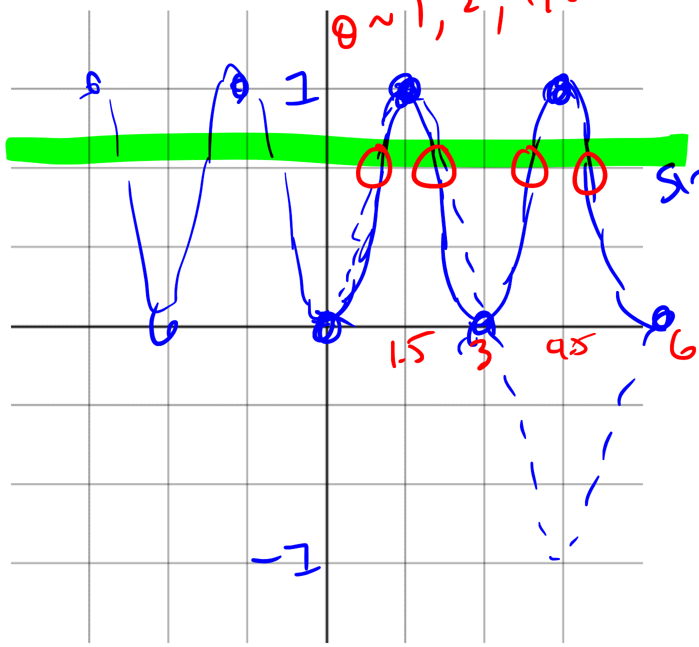
Practice: Find the general solution to the following

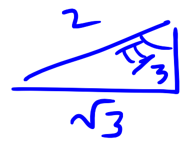
$$\csc^2\theta = \frac{4}{3}$$

$\iff \sin^2\theta = \frac{3}{4}$

$\sin\theta = \pm\frac{\sqrt{3}}{2}$

$\theta \sim 1, 2, 4, 5$

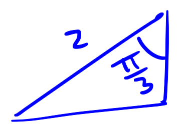




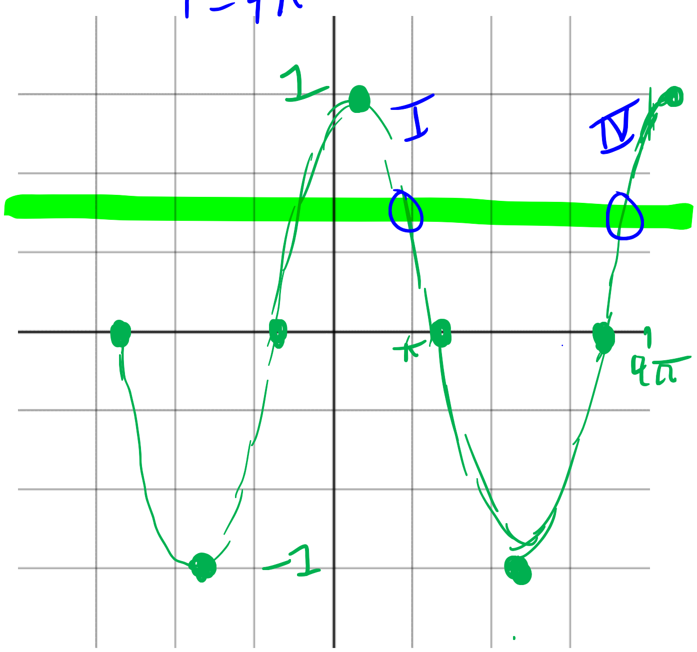
$$\boxed{\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} + 2\pi n}$$

Practice: Find the general solution to the following

$$\cos\left(\frac{1}{2}\left(\theta - \frac{\pi}{3}\right)\right) = \frac{1}{2}$$



$T = 4\pi$



$\frac{1}{2}\left(\theta - \frac{\pi}{3}\right) = \frac{\pi}{3}, \frac{5\pi}{3} + 2\pi n$

$\theta - \frac{\pi}{3} = \frac{2\pi}{3}, \frac{10\pi}{3} + 4\pi n$

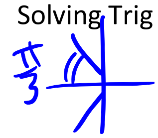
$\theta = \frac{3\pi}{3}, \frac{11\pi}{3} + 4\pi n$

$$\boxed{\begin{aligned} \theta_1 &= \pi + 4\pi n, \; n \in \mathbb{Z} \\ \theta_2 &= \frac{11\pi}{3} + 4\pi n \end{aligned}}$$

$\theta \sim \pi, 3.5\pi$

close

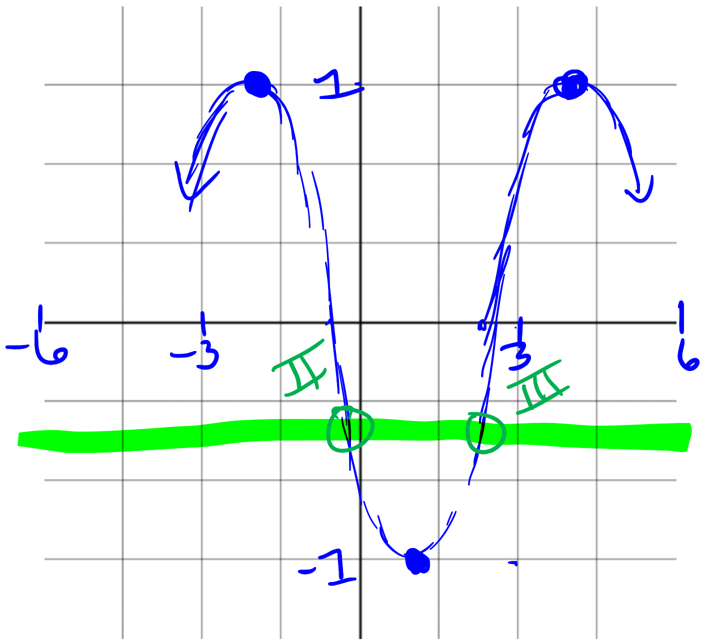
$11/3 = 3.\bar{6}$



Practice: Find the general solution to the following

$$-\sec\left(\frac{\pi}{3}(\theta+2)\right)=2$$

$T=\frac{2\pi}{\pi/3}=6$



$\theta \sim -0.5, 2$

$\Rightarrow \cos\left(\frac{\pi}{3}(\theta+2)\right)=-\frac{1}{2}$

$\frac{\pi}{3}(\theta+2)=\frac{2\pi}{3}, \frac{4\pi}{3}+2\pi n$

$\theta+2=2, 4+6n$

$\theta=0, 2+6n$

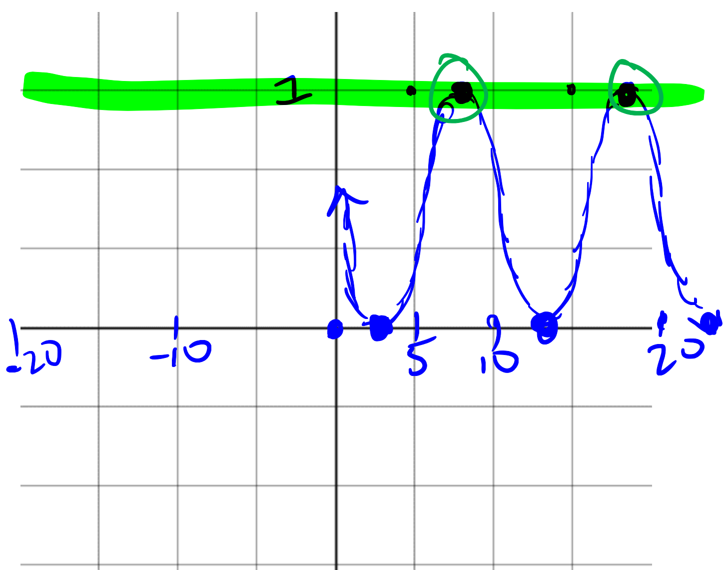
$$\boxed{\begin{aligned}\theta_1&=6n\\ \theta_2&=2+6n\end{aligned}\quad, n\in\mathbb{Z}}$$

Practice: Find the general solution to the following

$$\cos\theta\cdot\sin^2\left(\frac{\pi}{10}(\theta-3)\right)=\cos\theta$$

$T=\frac{2\pi}{\pi/10}=20$

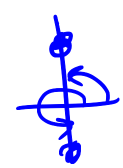
☆ $\cos\theta=0$



$\theta=8, 18$

$\sin^2\left(\frac{\pi}{10}(\theta-3)\right)=1$

$\sin\left(\frac{\pi}{10}(\theta-3)\right)=\pm1$



$\frac{\pi}{10}(\theta-3)=\frac{\pi}{2}+\pi n \quad n\in\mathbb{Z}$

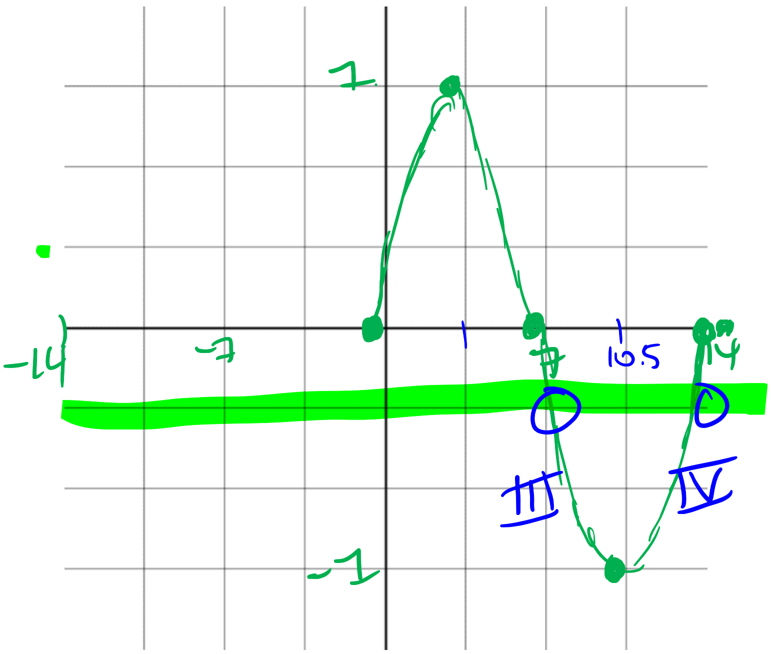
$\theta-3=5+10n$

$$\boxed{\theta=8+10n, n\in\mathbb{Z}}$$

Practice: Find the general solution to the following

$T = \frac{2\pi}{\pi/7} = 14$

$$\csc\left(\frac{\pi}{7}(\theta+1)\right) = -3$$



$\theta \sim 8, 12$

$\frac{1}{-3} = \sin\left(\frac{\pi}{7}(\theta+1)\right)$

$\frac{\pi}{7}(\theta+1) = \sin^{-1}\left(-\frac{1}{3}\right)$

$\frac{\pi}{7}(\theta+1) = 3.48, 5.94 + 2\pi n$

$\theta + 1 = 7.75, 13.24 + 14n$

$\theta = 6.75, 12.24 + 14n$

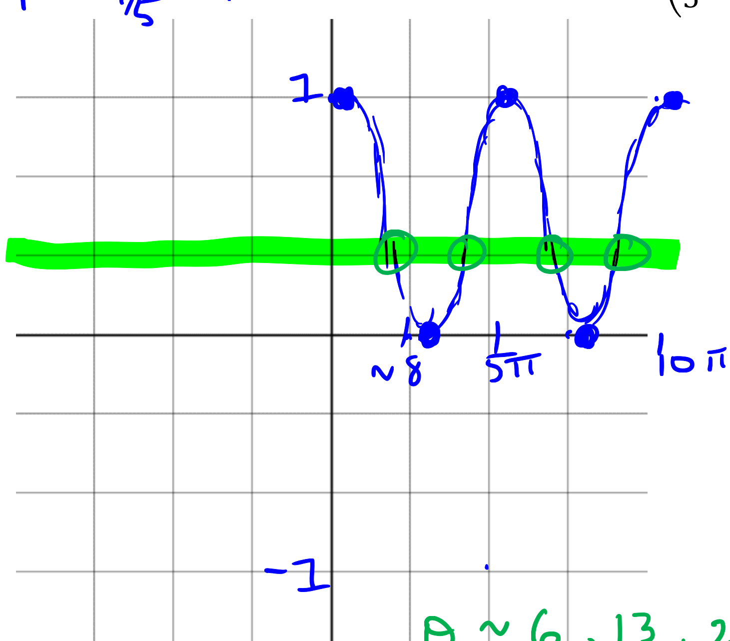
$\boxed{\begin{aligned}\theta_1 &= 6.75 + 14n \\ \theta_2 &= 12.24 + 14n\end{aligned}}$

Practice: Find the general solution to the following

$T = \frac{2\pi}{1/5} = 10\pi$

$$\cancel{4}\cos^2\left(\frac{1}{5}(\theta-2)\right) = \frac{1}{3}$$

$\sqrt{3}$, 1, not adj/hyp



$\cos\left(\frac{1}{5}(\theta-2)\right) = \pm\frac{1}{\sqrt{3}}$

$\frac{1}{5}(\theta-2) = 0.96, 2.19, 4.10, 5.32 + 2\pi n$

$\theta - 2 = 4.8, 10.95, 20.5, 26.6 + 10\pi n$

$\boxed{\theta = 6.8, 12.0, 22.5, 28.6 + 10\pi n}$

$\theta \sim 6, 13, 21, 25$

Suggested Practice Problems: 5.4 # 4, 5, 8, 11, 15-23	
Textbook Reading: page 266-273 Key Ideas page 274	
Next Class: Modelling Trig Functions	