Inverses as Reverse Mappings

Goal:

- Given a relation f, understand that the backwards relation is the same connection, but a different order.
- Can explain why inverses are reflections over the line $y = x$.
- Can determine an equation for the inverse of a function and can restrict the domain so the inverse is a function.

Terminology:

- Inverse
- One-to-one

When we look at the relationship a function f makes we know it takes a domain set to a range set

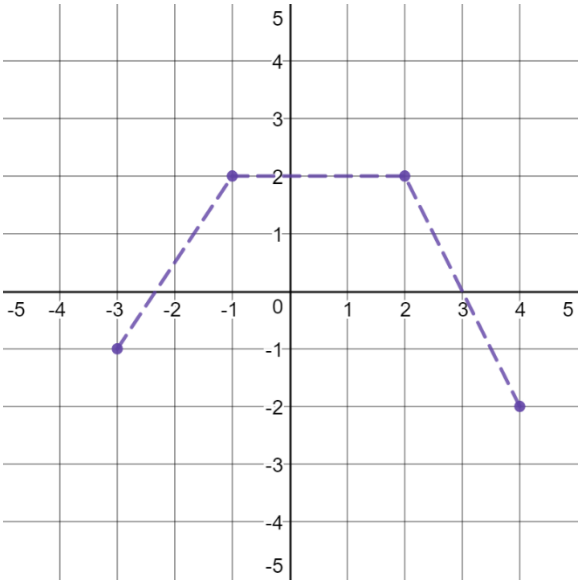
$$f: X \to Y$$
$$x \mapsto y$$
$$(x, y)$$

What we are interested in is the reverse relationship

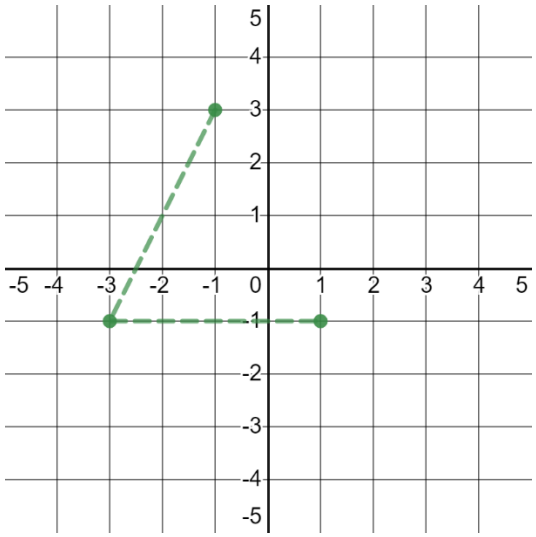
$$f^*: Y \to X$$
$$y \mapsto x$$
$$(y, x)$$

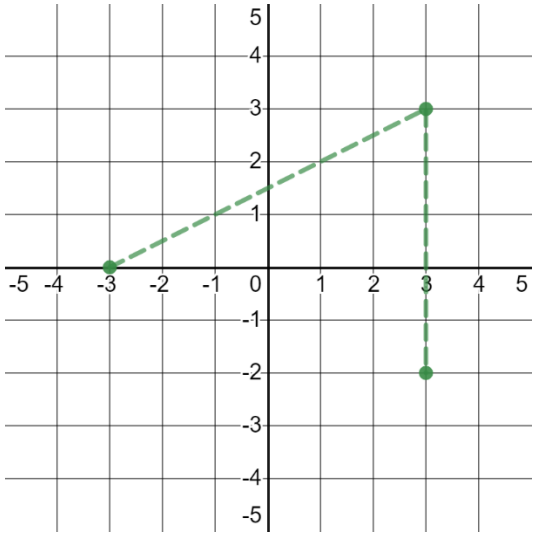
If we start with a function f, we can sketch the image of the inverse relation

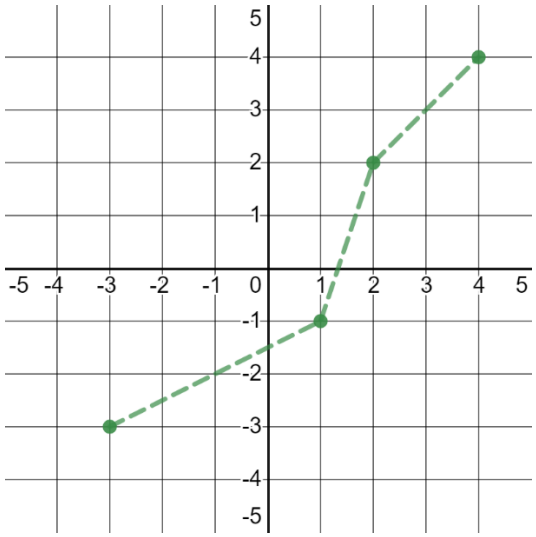
Example: Graph the inverse relation from the graph of f below



Practice: Graph the inverse relations of the following relations. What do you notice about orientation of the inverse image?







If the function was **one-to-one** to begin with, then the inverse relation will be a function.

To determine the equation of a function, $y = f(x)$, we want our output to be x and the input to be y.
In other words, we want to solve for x in the function $y = f(x)$.

Example: For the above function $f(x) = 2x - 3$ we want to solve for x.

Example: If g is one-to-one then find the inverse of $f(x) = 2g(x - 3) + 2$

Practice: Find the equation of the inverse of the following functions

$f(x) = \frac{x-1}{3}$

$f(x) = \frac{1}{4}x^3 + 3$

$f(x) = \frac{3}{2x-4} + 1$

$f(x) = \frac{g(0.5x)-1}{2}$

Example: If the function is not one-to-one, we need to make an adjustment to the domain.

$$f(x) = \left(\frac{x-2}{2}\right)^2 + 1$$

Practice: Find the inverse of the following function and restrict the domain so the inverse will be a function.

$f(x) = -2(x+3)^2 - 4$ $f(x) = (3x-6)^4 + 2$

Suggested Practice Problems: 1.4 page 51-55 # 1, 2, 4, 5, 9, 10, 12, 14, 15, 19-21, C1, C2
Textbook Reading: 1.3 page 46-50 Key Ideas on page 51
Next Class: Exponential function