Inverses as Reverse Mappings

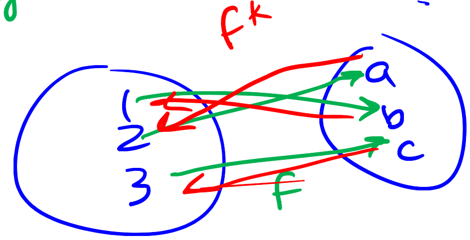
Goal:
- Given a relation f, understand that the backwards relation is the same connection, but a different order.
- Can explain why inverses are reflections over the line $y = x$.
- Can determine an equation for the inverse of a function and can restrict the domain so the inverse is a function.

Terminology:
- Inverse
- One-to-one

When we look at the relationship a function f makes we know it takes a domain set to a range set

Domain Range

$$f: X \to Y$$
$$x \mapsto y$$
$$(x, y)$$



What we are interested in is the reverse relationship

Star

$$f^*: Y \to X$$
$$y \mapsto x$$
$$(y, x)$$

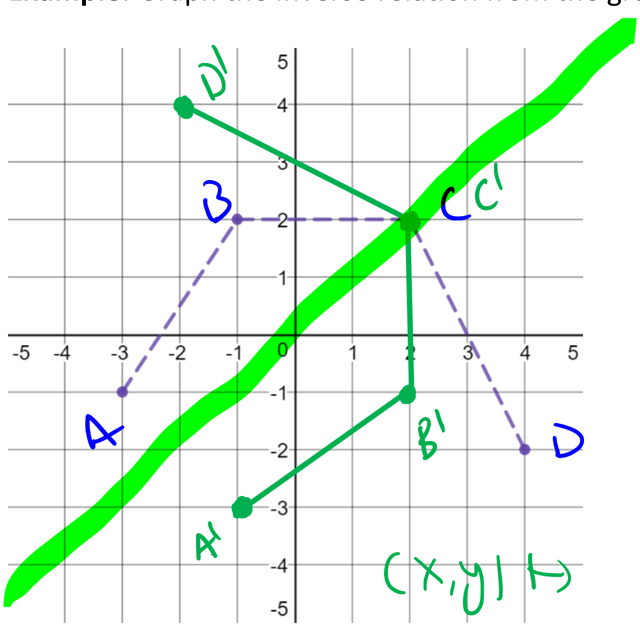
If we start with a function f, we can sketch the image of the inverse relation

$(x,y) \longmapsto (y,x)$

$(1,1) \mapsto (1,1)$ invariant point

$(5,3) \mapsto (3,5)$

Example: Graph the inverse relation from the graph of f below



$(x,y) \mapsto (y,x)$

$(-3,-1) \longmapsto (-1,-3)$

$(-1,2) \longmapsto (2,-1)$

$(2,2) \longmapsto (2,2)$

$(4,-2) \longmapsto (-2,4)$

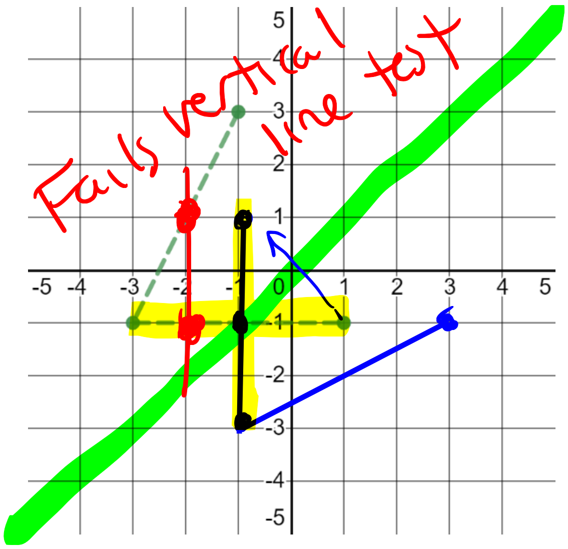
R_x then rotate 90° c.c

R_y ~~then rotate 90°~~ c.

$(x,y) \xrightarrow{R_y} (-x,y) \xrightarrow{90°} (y,x)$

$(a,b) \mapsto (b,a)$

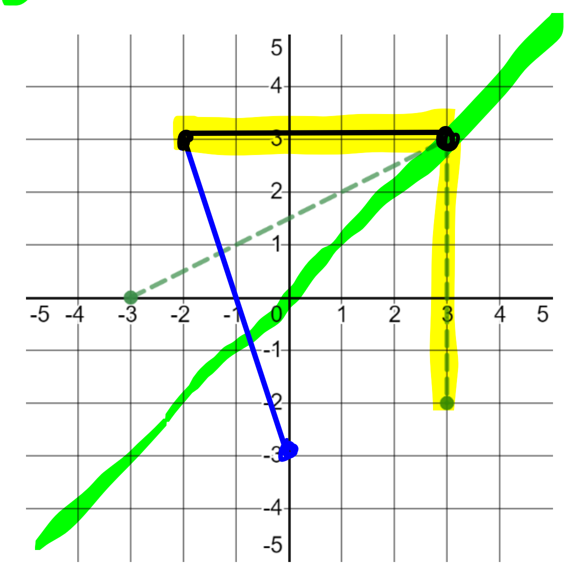
Practice: Graph the inverse relations of the following relations. What do you notice about orientation of the inverse image? & where are invariant points



horizontal lines become vertical

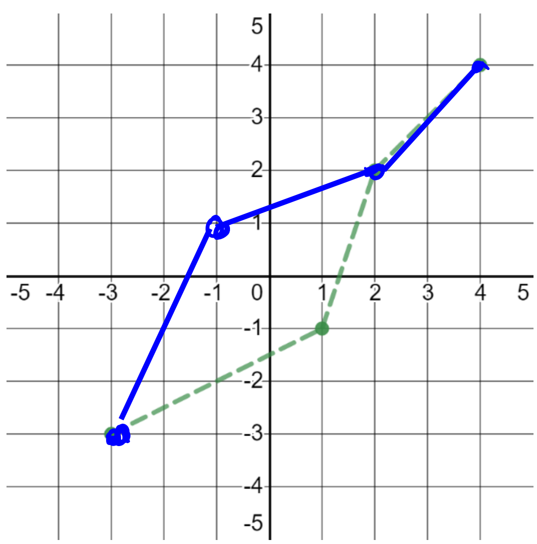
$-2 \mapsto 1$ & $-2 \mapsto -1$

NOT a function



and vice versa

Vertical lines become horizontal



Passes Horizontal line test

$\Rightarrow$ one-to-one

Then the inverse ~~is~~ pass vertical line test and is a function.

If the function was **one-to-one** to begin with, then the inverse relation will be a function.

$f^{-1}(x)$ is the inverse of f

f inverse of x

$f^{-1}(x) \neq \frac{1}{f(x)}$ BAD

To determine the equation of a function, $y = f(x)$, we want our output to be x and the input to be y. In other words, we want to solve for x in the function $y = f(x)$.

Example: For the above function $f(x) = 2x - 3$ we want to solve for x.

different

$y = 2x - 3$ f(x)

$x = 2y - 3$

$x + 3 = 2y$

$\boxed{\frac{x+3}{2} = y = f^{-1}(x)}$

$f^{-1}(x) = \frac{x+3}{2}$

$f(x) = 2x - 3$

1.) times by 2

2.) sub by 3

$f^{-1}(x) = \frac{x+3}{2}$

1.) add by 3

2.) div. by 2

Example: If g is one-to-one then find the inverse of $f(x) = 2g(x - 3) + 2$

$\boxed{y = 2g(x-3) + 2}$ f(x)

$x = 2g(y-3) + 2$

$x - 2 = 2g(y-3)$

$\frac{x-2}{2} = g(y-3)$ take g^{-1} of both sides

$g^{-1}\left(\frac{x-2}{2}\right) = y - 3$

$\boxed{g^{-1}\left(\frac{x-2}{2}\right) + 3 = y = f^{-1}(x)}$

Practice: Find the equation of the inverse of the following functions

$f(x) = \frac{x-1}{3}$

$y = \frac{x-1}{3}$

$x = \frac{y-1}{3}$

$3x = y-1$

$y = 3x+1 = f^{-1}(x)$

$f(x) = \frac{1}{4}x^3 + 3$

$y = \frac{1}{4}x^3 + 3$

$x = \frac{1}{4}y^3 + 3$

$x - 3 = \frac{1}{4}y^3$

$4(x-3) = y^3$

$\sqrt[3]{4(x-3)} = f^{-1}(x)$

$f(x) = \frac{3}{2x-4} + 1$

$x = \frac{3}{2y-4} + 1$

$\frac{x-1}{1} = \frac{3}{2y-4}$

$\frac{1}{x-1} = \frac{2y-4}{3}$

$\frac{3}{x-1} = 2y-4$

$\frac{3}{x-1} + 4 = 2y$

$y = \frac{3}{2(x-1)} + 2 = f^{-1}(x)$

$f(x) = \frac{g(0.5x)-1}{2}$

$x = \frac{g(\frac{1}{2}y)-1}{2}$

$2x = g(\frac{1}{2}y) - 1$

$2x+1 = g(\frac{1}{2}y)$

$g^{-1}(2x+1) = \frac{1}{2}y$

$y = 2g^{-1}(2x+1) = f^{-1}(x)$

right 2 → exp by 2 ??

Example: If the function is not one-to-one, we need to make an adjustment to the domain.

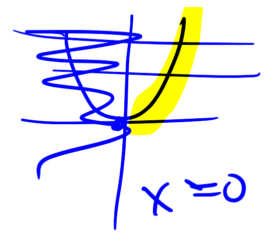
$$f(x) = \left(\frac{x-2}{2}\right)^2 + 1$$

right 2

$g(x) = x^2$

$g\left(\frac{x-2}{2}\right) + 1$

$(x, y) \mapsto (2x + 2, y + 1)$



$x = 0$

positive half

let $x \geq 2$

$y = \left(\frac{x-2}{2}\right)^2 + 1$

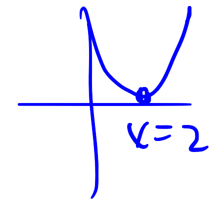
$x = \left(\frac{y-2}{2}\right)^2 + 1$

$x - 1 = \left(\frac{y-2}{2}\right)^2$

$+\sqrt{x-1} = \frac{y-2}{2}, \; y \geq 2$

$2\sqrt{x-1} = y - 2$

$\boxed{2\sqrt{x-1} + 2 = f^{-1}(x)}$



$x = 2$

$x \mapsto \frac{x+6}{3} = \frac{x}{3} + 2$

Practice: Find the inverse of the following function and restrict the domain so the inverse will be a function.

$f(x) = -2(x+3)^2 - 4$

$x = -3, \; x \leq -3$

$y = -2(x+3)^2 - 4$

$x = -2(y+3)^2 - 4$

$x + 4 = -2(y+3)^2$

$\frac{x+4}{-2} = (y+3)^2$

$-\sqrt{\frac{x+4}{-2}} = y + 3, \; y \leq -3$ −ve

$-\sqrt{\frac{x+4}{-2}} - 3 = f^{-1}(x)$

$f(x) = (3x-6)^4 + 2$

$3x - 6 = 0 \quad 3x = 6 \quad x = 2$

$x \geq 2$

$y = (3x-6)^4 + 2$

$x = (3y-6)^4 + 2$

$x - 2 = (3y-6)^4$

$+\sqrt[4]{x-2} = 3y - 6, \; y \geq 2$ +ve

$\boxed{\frac{\sqrt[4]{x-2} + 6}{3} = f^{-1}(x)}$

Suggested Practice Problems: 1.4 page 51-55 # 1, 2, 4, 5, 9, 10, 12, 14, 15, 19-21, C1, C2	
Textbook Reading: 1.3 page 46-50	
Key Ideas on page 51	
Next Class: Exponential function	