Inverses as Reverse Mappings

Goal:

- Given a relation f, understand that the backwards relation is the same connection, but a different order.
- Can explain why inverses are reflections over the line $y = x$.
- Can determine an equation for the inverse of a function and can restrict the domain so the inverse is a function.

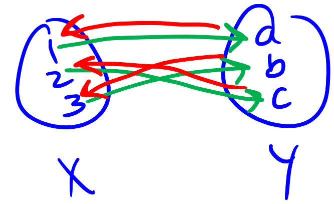
Terminology:

- Inverse
- One-to-one

When we look at the relationship a function f makes we know it takes a domain set to a range set

Domain Range

$$f: X \to Y$$
$$x \mapsto y$$
$$(x, y)$$



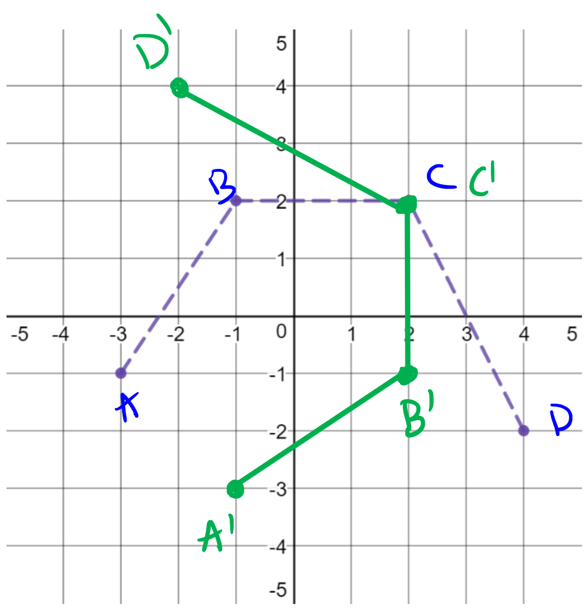
What we are interested in is the reverse relationship

$$f^*: Y \to X$$
$$y \mapsto x$$
$$(y, x)$$

If we start with a function f, we can sketch the image of the inverse relation

$(x,y) \mapsto (y,x)$

Example: Graph the inverse relation from the graph of f below



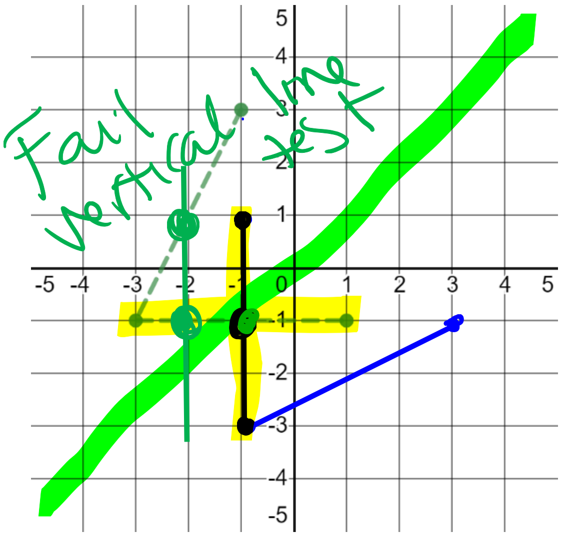
$(-3,-1) \mapsto (-1,-3)$

$(-1,2) \mapsto (2,-1)$

$(2,2) \mapsto (2,2)$ INVARIENT

$(4,-2) \mapsto (-2,4)$

Practice: Graph the inverse relations of the following relations. What do you notice about orientation of the inverse image? where are the invariant points

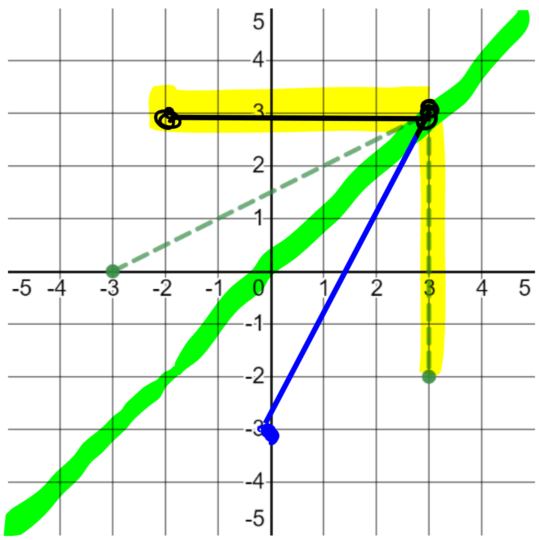


$(x_1,x_2) \mapsto (x_2,x_1)$

horizontal lines become vertical

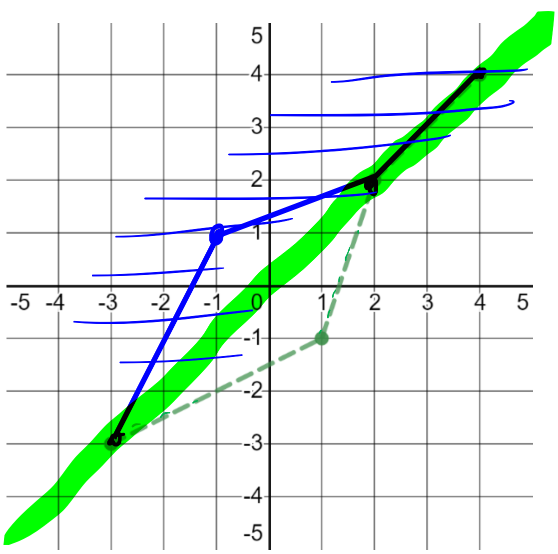
$-2 \mapsto -1$ & $-2 \mapsto 1$

NOT a function



vertical lines become horizontal

passes horizontal line test $\Rightarrow$ Then its one-to-one



$(4,4) \mapsto (4,4)$

$(2,2) \mapsto (2,2)$

$(1,-1) \mapsto (-1,1)$

$(-3,-3) \mapsto (-3,-3)$

☆ inverse is a function

same reflection?

no stretches

If the function was **one-to-one** to begin with, then the inverse relation will be a function.

$f^{-1}(x)$ is the inverse function of f

f inverse of x

To determine the equation of a function, $y = f(x)$, we want our output to be x and the input to be y. In other words, we want to solve for x in the function $y = f(x)$.

Example: For the above function $f(x) = 2x - 3$ we want to solve for x.

$\boxed{y = 2x - 3}$ $f(x)$

f { 1.) times by 2
2.) sub by 3

$x = 2y - 3$

f^{-1} { 1.) add 3
2.) div. by 2

$x + 3 = 2y$

$\frac{x+3}{2} = y \Rightarrow f^{-1}(x) = \frac{x+3}{2}$

$f(1) = -1 \qquad f^{-1}(-1) = 1$

$1 \xrightarrow{f} -1$

$1 \xleftarrow{f^{-1}} -1$

Example: If g is one-to-one then find the inverse of $f(x) = 2g(x - 3) + 2$

$y = 2g(x-3) + 2$

$x = 2g(y-3) + 2$

$x - 2 = 2g(y-3)$

$\frac{x-2}{2} = g(y-3)$

$g^{-1}\left(\frac{x-2}{2}\right) = y - 3$

$g^{-1}\left(\frac{x-2}{2}\right) + 3 = f^{-1}(x)$

Practice: Find the equation of the inverse of the following functions

$f(x) = \frac{x-1}{3}$

$f(x) = \frac{1}{4}x^3 + 3$

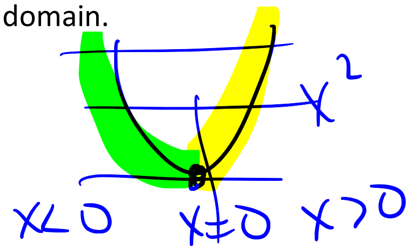
see other copy

$f(x) = \frac{3}{2x-4} + 1$

$f(x) = \frac{g(0.5x)-1}{2}$

Example: If the function is not one-to-one, we need to make an adjustment to the domain.

$$f(x) = \left(\frac{x-2}{2}\right)^2 + 1$$



$\frac{x-2}{2} = 0 \Rightarrow x = 2$

$\boxed{x > 2}$

$y = \left(\frac{x-2}{2}\right)^2 + 1$

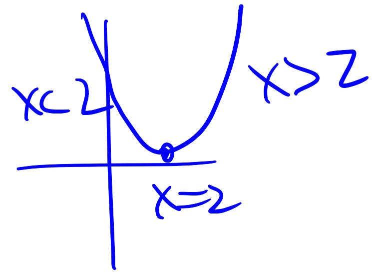
$x = \left(\frac{y-2}{2}\right)^2 + 1$

$x - 1 = \left(\frac{y-2}{2}\right)^2$

$+\sqrt{x-1} = \frac{y-2}{2}, \ y > 2$

$2\sqrt{x-1} + 2 = y$

$\boxed{f^{-1}(x) = 2\sqrt{x-1} + 2}$



Practice: Find the inverse of the following function and restrict the domain so the inverse will be a function.

$f(x) = -2(x+3)^2 - 4$

$x > -3$

$x = -2(y+3)^2 - 4$

$\frac{x+4}{-2} = (y+3)^2$

$+\sqrt{\frac{x+4}{-2}} = \overbrace{y+3}^{+ve}, \ y > -3$

$\boxed{\sqrt{\frac{x+4}{-2}} - 3 = f^{-1}(x)}$

$f(x) = (3x-6)^4 + 2$

$x < 2$

$3x - 6 = 0 \quad x = 2$

$x = (3y-6)^4 + 2$

$-\sqrt[4]{x-2} = 3y - 6, \ y < 2$

$-\sqrt[4]{x-2} + 6 = 3y$

$\frac{6 - \sqrt[4]{x-2}}{3} = y = f^{-1}(x)$

Suggested Practice Problems: 1.4 page 51-55 # 1, 2, 4, 5, 9, 10, 12, 14, 15, 19-21, C1, C2	
Textbook Reading: 1.3 page 46-50	Key Ideas on page 51
Next Class: Exponential function	