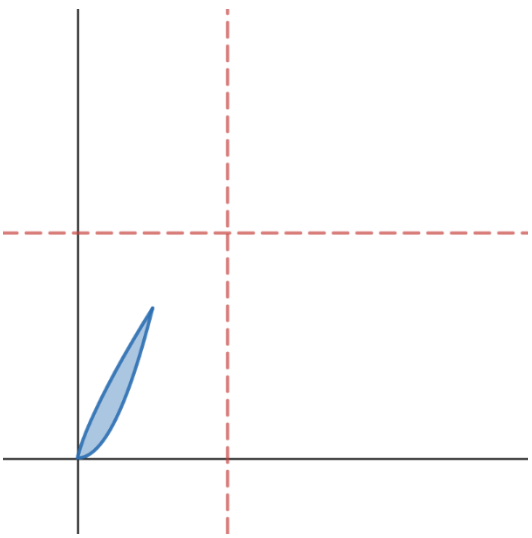
Volumes Part 2: Washers and Shells

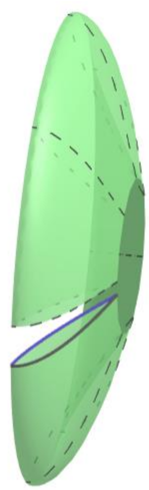
Goal: • Can determine the volume of solids after rotation around any line. • Can use washers and discs to find the volume of a shape. • Can determine the volume of solids after rotation using shells
Terminology: • None

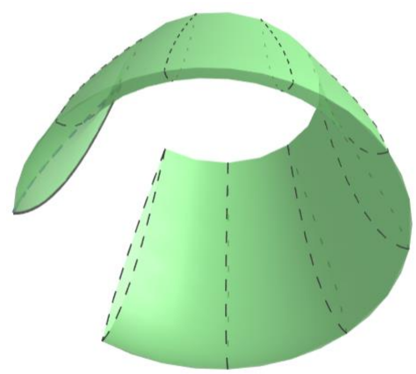
Volume using washers:
The region bound between the two one-to-one functions $y = f(x)$ and $y = g(x)$ is rotated around

a. The line $x = a$
b. The line $y = b$

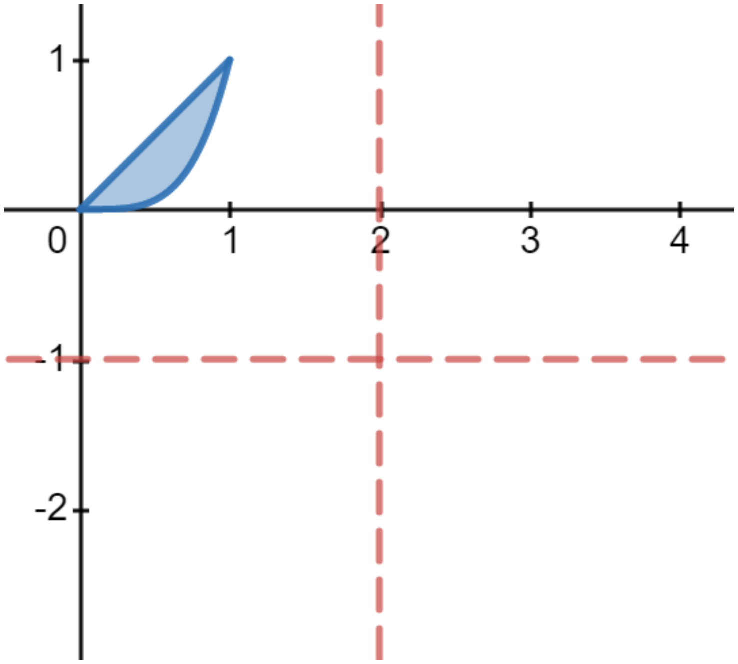
Write the expression for the volume formed.The curves intersect at the origin and the point (x_0, y_0) and $\forall\, x \in (0, x_0)$

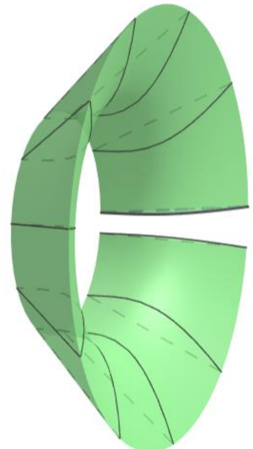


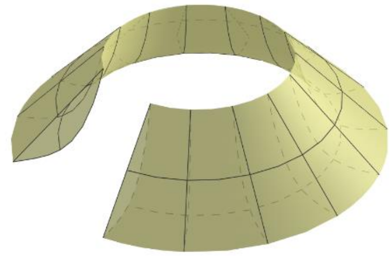




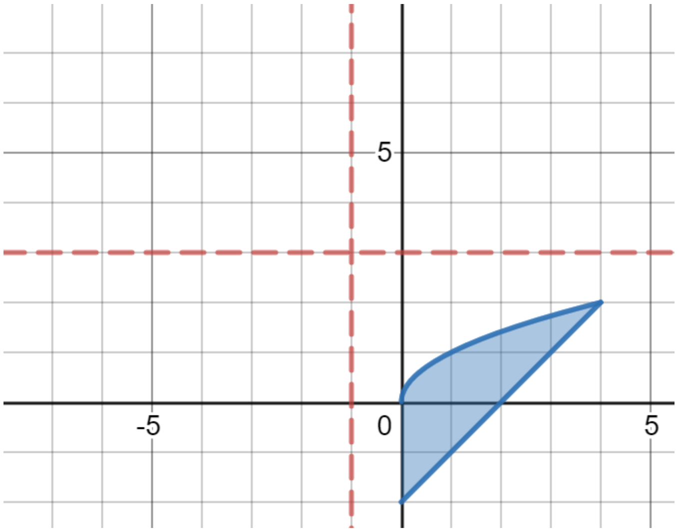
Practice: The region bound between the curves $y = x$ and $y = x^4$ is rotated about the line $x = 2$ and $y = -1$. What are the volumes?



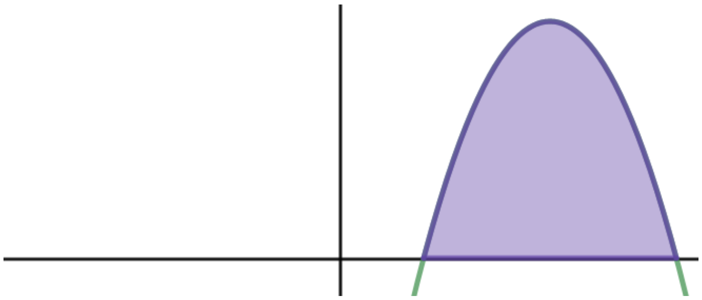
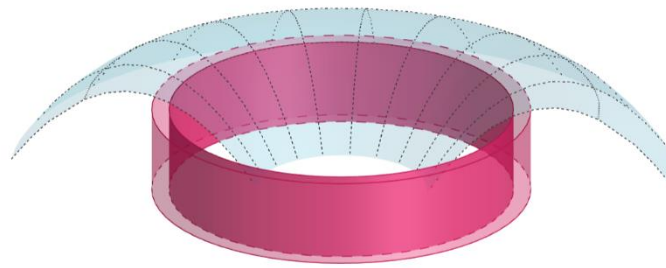




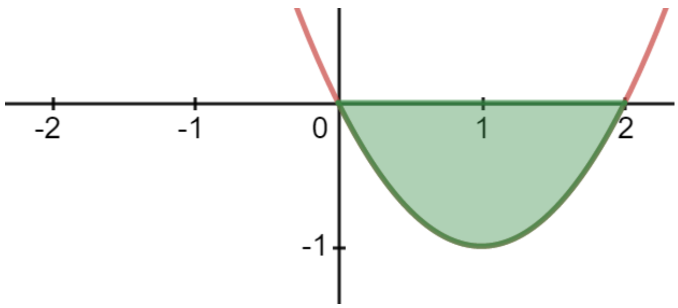
Practice: Determine the volume of the shape made by rotating the region bound between $y = \sqrt{x}$ and $y = x - 2$ and the y-axis after rotating it around the line $x = -1$ and $y = 3$

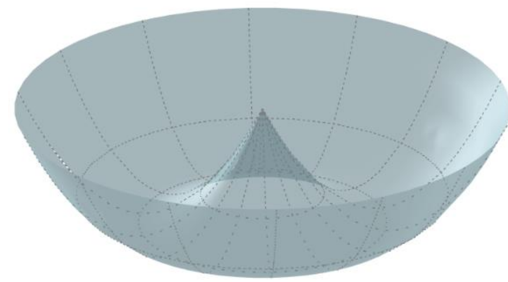


Volume using Shells:
Given a region bound between the function $y = f(x)$ (not 1-to-1) and the x-axis on the interval $x \in [a, b]$ and rotated around the y-axis, define a Riemann sum using cylinders whose limit is the desired volume.

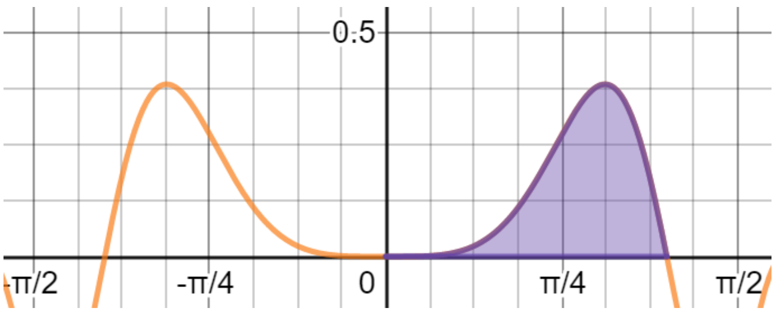
Practice: Determine the volume formed when the region bound between the curve $y = x^2 - 2x$ and the x-axis on the interval $x \in [0, 2]$ is rotated about the y-axis

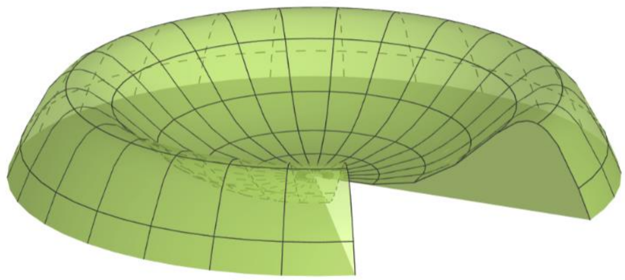




Take the same area and rotate it about the line $x = -1$ instead. What is the new volume?

Practice: Determine the volume made when the region enclosed between $y = \sin^2 x^2 \cdot \cos x^2$ and the x-axis on the interval $x \in \left[0, \sqrt{\frac{\pi}{2}}\right]$ after being rotated about the y-axis.





Practice Problems: 7.3 # 21-26, 39-53	
Textbook Readings: 7.3 page 386-389	
Workbook Practice: page 338-349	
Next Class: Differential Equations	