Volumes Part 2: Washers and Shells

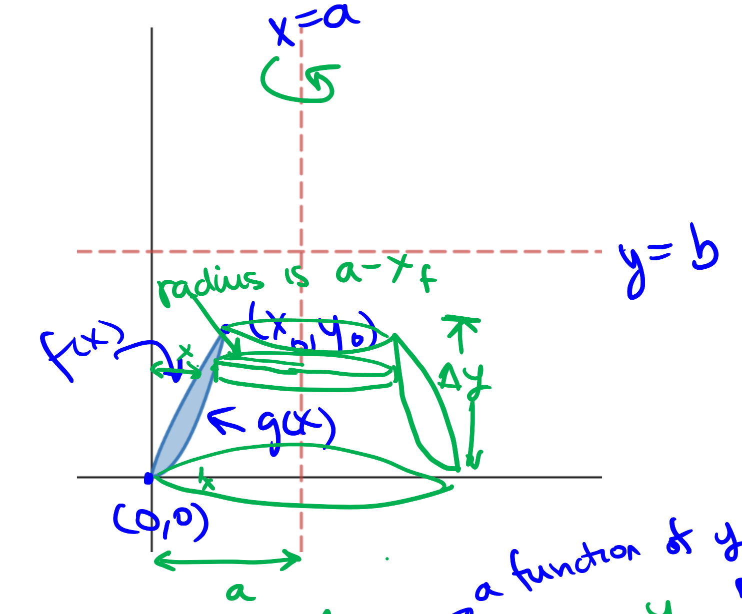
Goal: • Can determine the volume of solids after rotation around any line. • Can use washers and discs to find the volume of a shape. • Can determine the volume of solids after rotation using shells
Terminology: • None

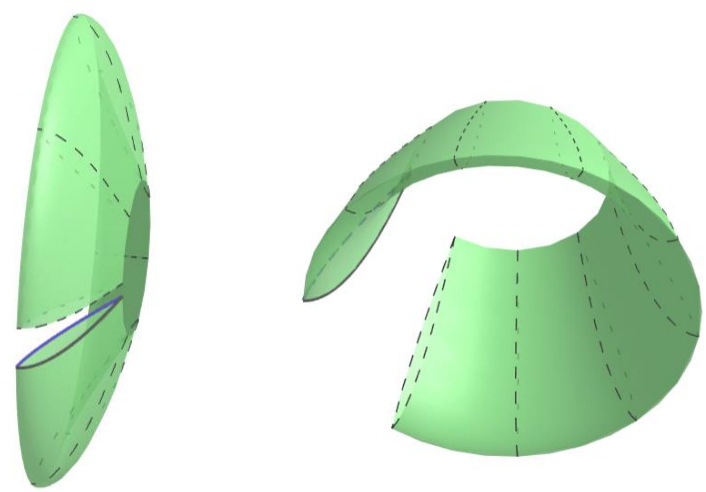
Volume using washers:

The region bound between the two one-to-one functions $y = f(x)$ and $y = g(x)$ is rotated around

a. The line $x = a$
b. The line $y = b$

Write the expression for the volume formed.The curves intersect at the origin and the point (x_0, y_0) and $\forall\, x \in (0, x_0)$





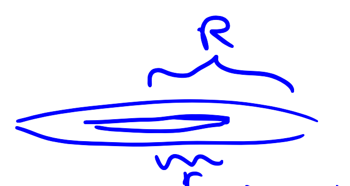
a function of y

$y=f(x)$
$x=f^{-1}(y)$

about $x=a$

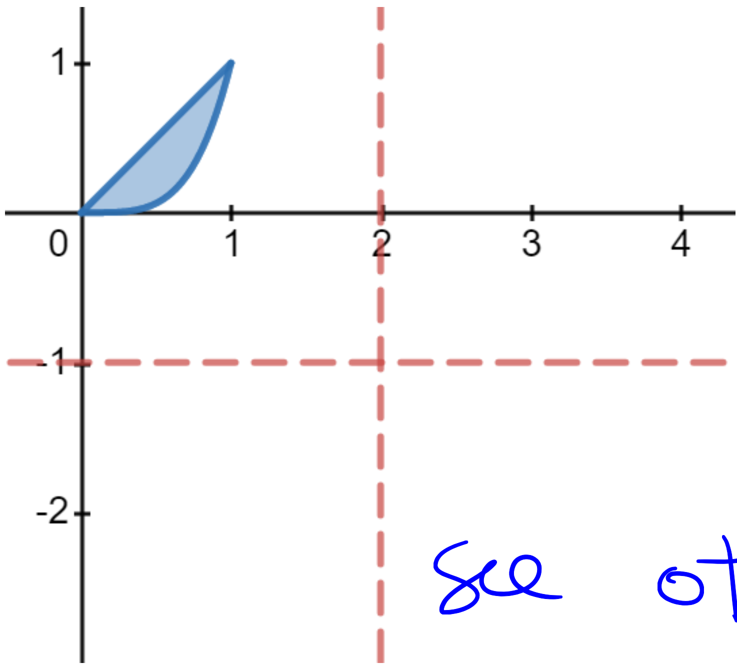
$$V = \int_0^{y_0} \pi(a-x_f)^2\,dy - \int_0^{y_0} \pi(a-x_s)^2\,dy$$

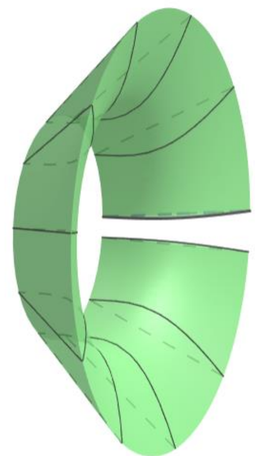
$$= \pi \int_0^{y_0} \Big(\underbrace{(a-f^{-1}(y))^2}_{\text{large Radius}} - \underbrace{(a-g^{-1}(y))^2}_{\text{small radius}} \Big)\,dy$$

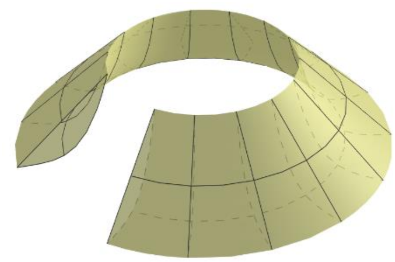


$A = \pi(R^2 - r^2)$

Practice: The region bound between the curves $y = x$ and $y = x^4$ is rotated about the line $x = 2$ and $y = -1$. What are the volumes?







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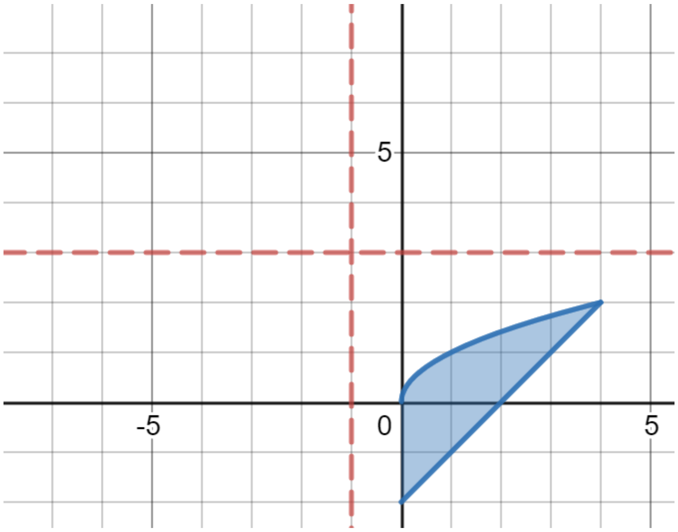
Practice: Determine the volume of the shape made by rotating the region bound between $y = \sqrt{x}$ and $y = x - 2$ and the y-axis after rotating it around the line $x = -1$ and $y = 3$



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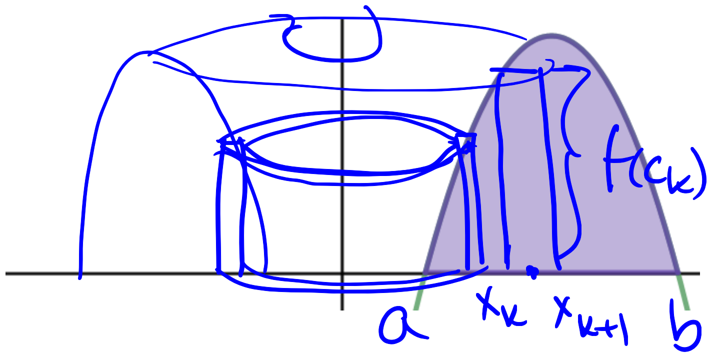
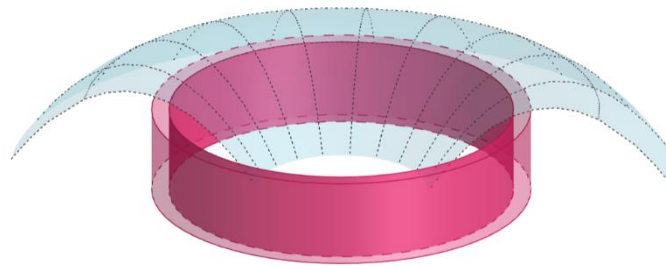
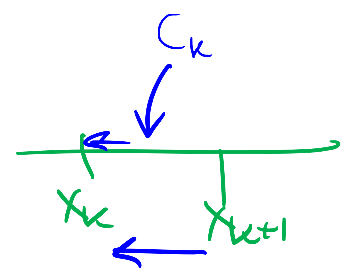
Volume using Shells:
Given a region bound between the function $y = f(x)$ (not 1-to-1) and the x-axis on the interval $x \in [a, b]$ and rotated around the y-axis, define a Riemann sum using cylinders whose limit is the desired volume.





$$V_k = \pi\left(x_{k+1}^2 \cdot f(c_k) - x_k^2 \cdot f(c_k) \right)$$

$$= \pi f(c_k) \underbrace{(x_{k+1} - x_k)}_{\Delta x}(x_{k+1} + x_k)$$



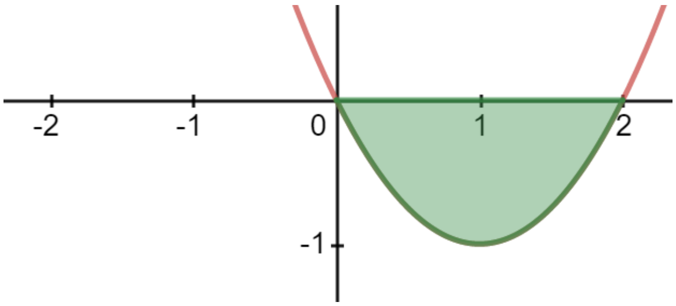
$$V = \lim_{n\to\infty} \sum \pi f(c_k)(x_{k+1} + x_k)\Delta x$$

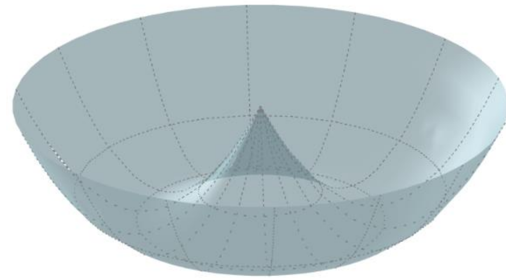
$$\int_a^b \pi f(x)(2x)\,dx$$

$$= \int_a^b \underbrace{2\pi x}_{\text{circumference}} \cdot \underbrace{f(x)}_{\text{height}}\,dx$$

$\Rightarrow$ SA of a cylindrical shell.

Practice: Determine the volume formed when the region bound between the curve $y = x^2 - 2x$ and the x-axis on the interval $x \in [0, 2]$ is rotated about the y-axis





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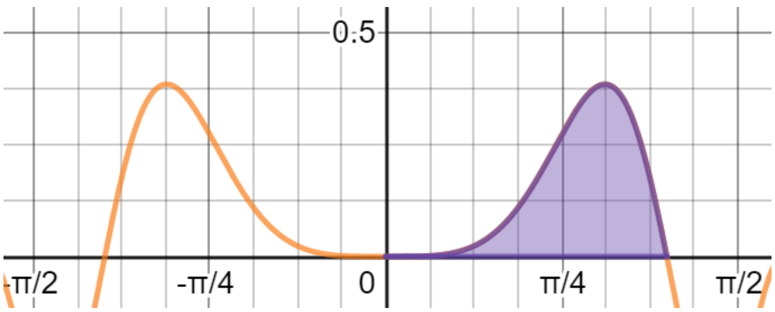
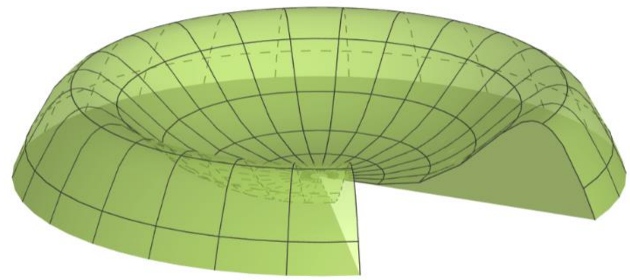
Take the same area and rotate it about the line $x = -1$ instead. What is the new volume?

Practice: Determine the volume made when the region enclosed between $y = \sin^2 x^2 \cdot \cos x^2$ and the x-axis on the interval $x \in \left[0, \sqrt{\frac{\pi}{2}}\right]$ after being rotated about the y-axis.





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Practice Problems: 7.3 # 21-26, 39-53
Textbook Readings: 7.3 page 386-389
Workbook Practice: page 338-349
Next Class: Differential Equations