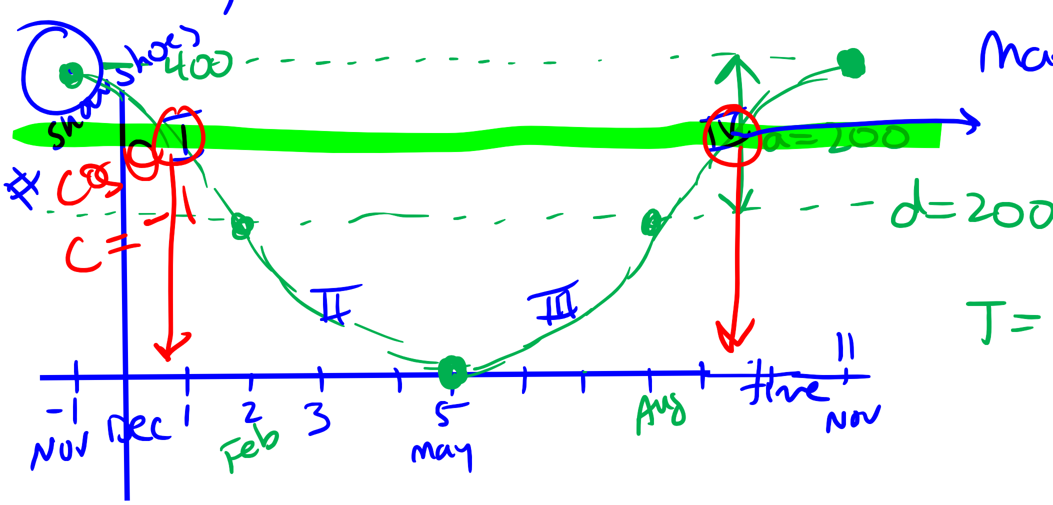
Modelling Trig Equations

Goal:
• Can make trig equations to model situations.
Terminology:
• None

Example: Sales of snowshoes at MEC are seasonal with the most occurring in November at 400 units sold and basically none are sold 6 months later. Determine an equation for the sales in terms of time and determine when 300 units are sold. (May)

time , # of snowshoes

let Nov be $t=-1$
Dec be $t=0$
May $t=5$

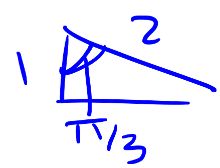


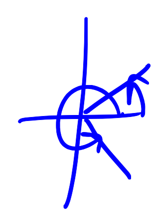
$T=12 \Rightarrow b = \frac{2\pi}{12} = \frac{\pi}{6}$

$$S(m) = 200\cos\frac{\pi}{6}(m+1) + 200$$

$S(m) = 300$ when $m \sim 1$, 10
Jan, Oct

$$\Rightarrow 300 = 200\cos\frac{\pi}{6}(m+1) + 200$$





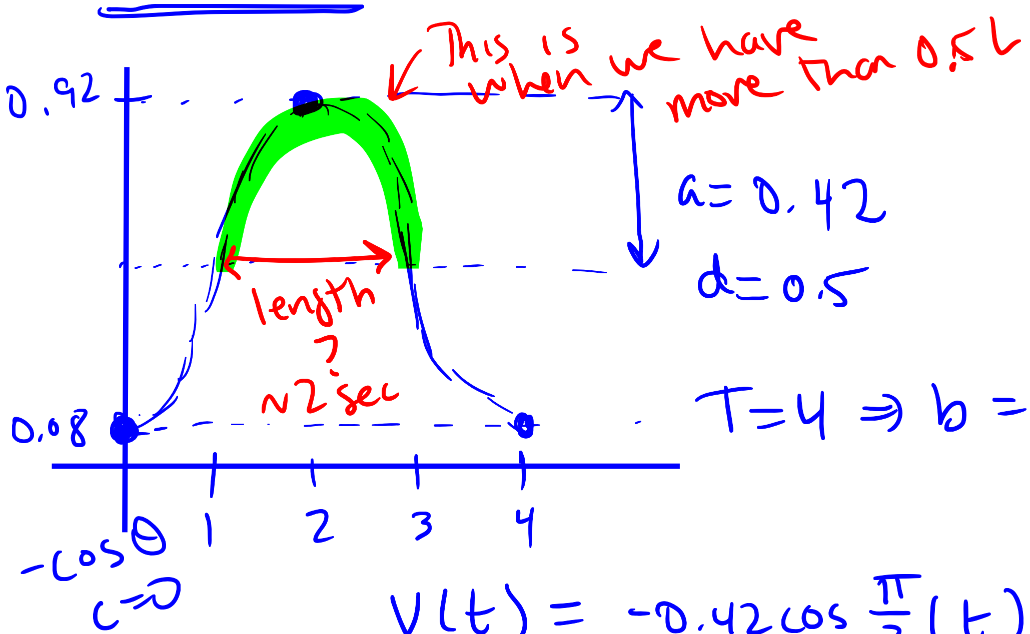
$$\frac{100}{200} = \cos\frac{\pi}{6}(m+1)$$

$$\Rightarrow \frac{\pi}{6}(m+1) = \frac{\pi}{3}, \frac{5\pi}{3} + 2\pi n$$

$$m+1 = 2, 10 + 12n$$

$$m = 1, 9 + 12n \quad \Rightarrow \text{January \& September}$$

Example: A normal adult breathes in and exhales about 0.84 L of air every 4 seconds. When they exhale, the minimal amount in the lungs of 0.08 L. Determine an equation for the volume of air in their lungs at time t. Determine how long in one cycle they will have more than 0.5 L of air in their lungs.



$a = 0.42$

$d = 0.5$

$T = 4 \Rightarrow b = \frac{2\pi}{4} = \frac{\pi}{2}$

$$V(t) = -0.42\cos\frac{\pi}{2}(t) + 0.5$$

$$0.5 = -0.42\cos\frac{\pi}{2}t + 0.5$$

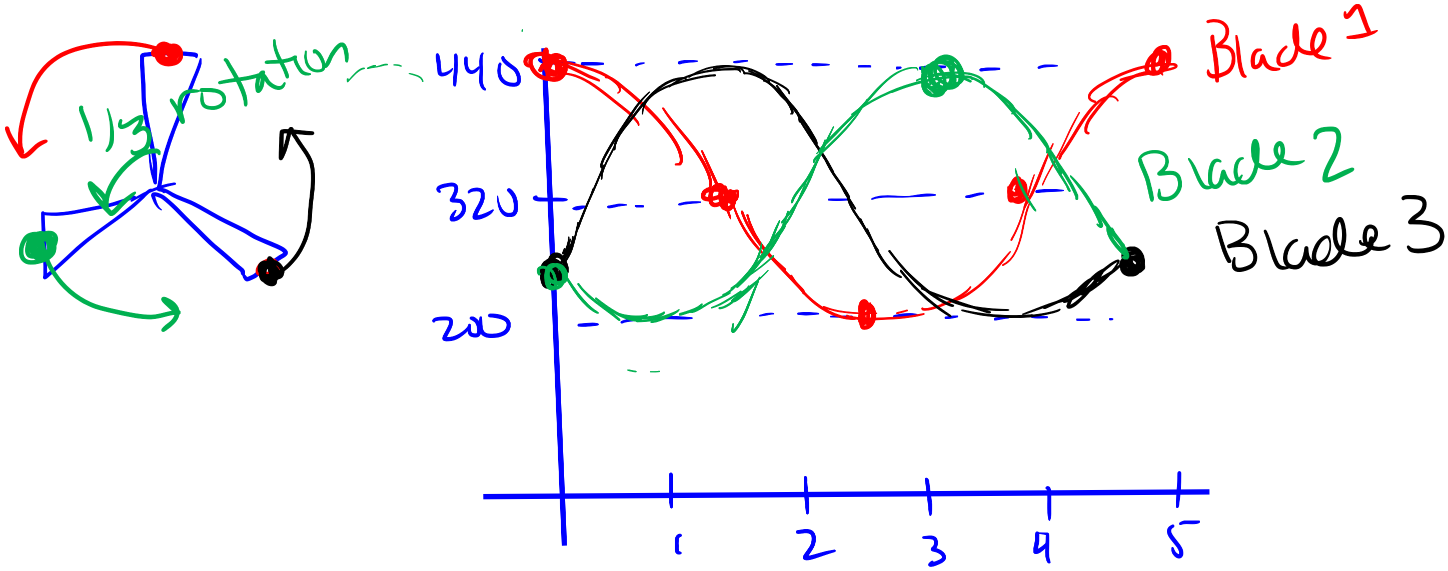
$$0 = \cos\frac{\pi}{2}t$$

$$\Rightarrow \frac{\cancel{\pi}}{2}t = \frac{\cancel{\pi}}{2} + \pi n$$

$$t = 1 + 2n$$

so @ 1 second and 3 second they have 0.5 L $\Rightarrow$ in the 4 second period they have more than 0.5 L for 2 seconds

Example: A windmill has three blades that are 120 feet long and the propeller spins once every 5 seconds. If the center of the windmill blades are 320 feet above the ground and the blades are spaced evenly apart determine an equation for the height of each blade above the ground at time t.



$T = 5 \Rightarrow b = \frac{2\pi}{5}$

$a = 120$

$d = 320$

Blade 1 $c = 0$, $\cos\theta$

$$h(t) = 120\cos\frac{2\pi}{5}t + 320$$

Blade 2 is $\frac{1}{3}T$ in front of Blade 1

$\Rightarrow$ shift Blade 1 back $\frac{T}{3}$ ($c = -5/3$)

$$h(t) = 120\cos\frac{2\pi}{5}(t + 5/3) + 320$$

Blade 3 is $\frac{1}{3}T$ behind Blade 1

$\Rightarrow$ shift Blade 1 ahead $\frac{T}{3}$ ($c = +5/3$)

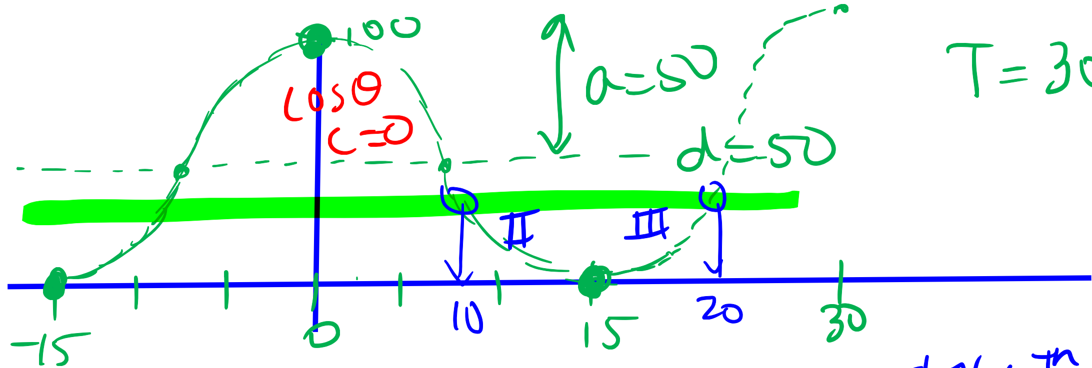
$$h(t) = 120\cos\frac{2\pi}{5}(t - 5/3) + 320$$

Example: The phase of the moon follows a sinusoidal path. On October 16, 2020 there was a new moon (it was be 0% visible) and on October 31, 2020 there will be a full moon (100% visible). Make an equation for the days in NOVEMBER the determine how much the moon is visible. Use it to determine the days when the moon is 30% visible.

$t = 1 \equiv$ Nov 1st

$t = 0 \equiv$ Oct 31st

$t = -15 = $ Oct 16th



$T = 30 \Rightarrow b = \frac{2\pi}{30} = \frac{\pi}{15}$

$d \sim 10^{th}, 20^{th}$ of Nov

$$V(d) = 50\cos\frac{\pi}{15}d + 50$$

$$30 = 50\cos\frac{\pi}{15}d + 50$$

$$-\frac{2}{5} = \cos\frac{\pi}{15}d$$

$$\arccos(-2/5) = \pi/15\, d$$

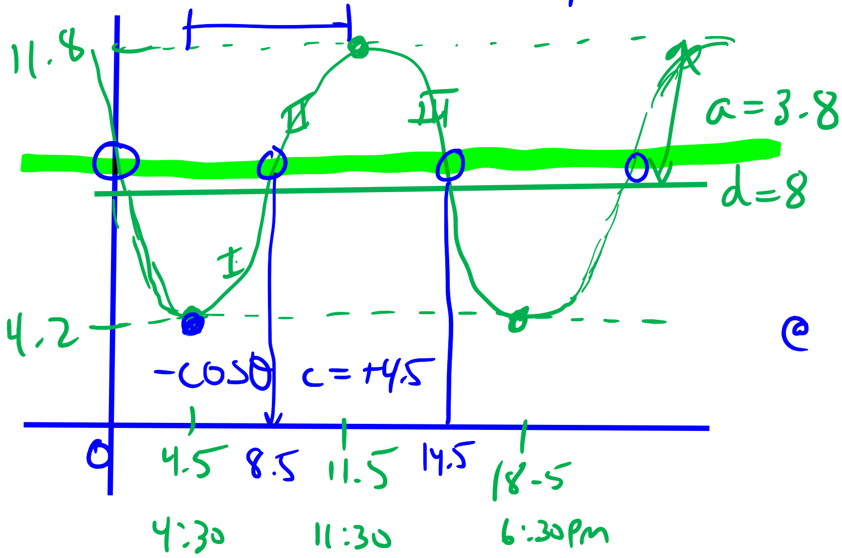
$$1.98,\ 4.30 + 2\pi n = \frac{\pi}{15}d$$

$$\Rightarrow d = 9.95,\ 20.5 + 30n$$

Between the 9th and 10th & 20th to 21st the moon will have 30% visibility.

Example: A low tide of 4.2 m in White Rock occurs at 4:30 AM and the next high tide of 11.8 m occurs at 11:30 AM. Determine an equation for the height of the tide at time t in hours. Determine the times in that day when the tide is 9 m.

$7 \Rightarrow T = 14 \Rightarrow b = \frac{2\pi}{14} = \frac{\pi}{7}$



@ $t \sim 0, 8.5, 14.5$

midnight, 8:30 AM, 2:30PM

about

$h(t) = -3.8\cos\frac{\pi}{7}(t-4.5) + 8$

$9 = -3.8\cos\frac{\pi}{7}(t-4.5) + 8$

$\frac{-1}{3.8} = \cos\frac{\pi}{7}(t-4.5)$

$\arccos\left(-\frac{1}{3.8}\right) = \frac{\pi}{7}(t-4.5)$

$\Rightarrow 1.84, 4.45 + 2\pi n = \frac{\pi}{7}(t-4.5)$

$\Rightarrow t = 8.6, 14.4 + 14n$

In one day $t = 0.4, 8.6, 14.4, 22.6$

@ 12:24AM, 8:36AM
2:24PM, 10:36PM

Suggested Practice Problems: 5.4 # 4, 5, 8, 11, 15-23	
Textbook Reading: page 266-273 Key Ideas page 274	
Next Class: Trig Identities	