Solving Exponents and Logarithms Algebraically

Goal: • Can solve equations involving logs using algebra and remember to respect the domain of log functions. • Can model problems using the natural log thinking of the relationship to the exponential function
Terminology: • Extraneous Solutions

When solving log equations, we need to remember to respect the domain of the original expression

Example: Solve for x

$$-3 + \log_2 x = -\log_4(x+1)^2$$

Practice: Solve for x

$$\log_3\big((x+3)(x-4)\big) = 6\log_{27}(x+3) + 1$$

We want to practice modelling exponential and log equations and solving by changing our bases to 10 and e.

Practice: The population of the Greater Vancouver Area last year (2019) was 2.5 million. It is expected to grow at an annual rate of 0.95% for the next few decades. Write a function for the population at year t using e as the base.

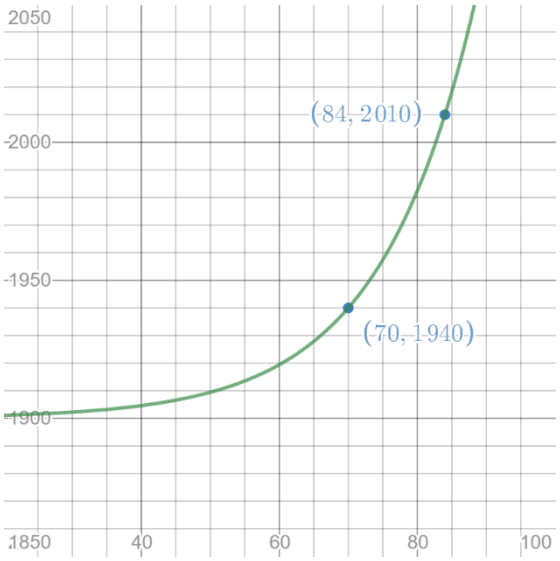
Write the equation to the inverse that outputs the year given the current population.

Determine the year the population of Vancouver will reach 4 million. Determine the year when then population of Vancouver was 1 million.

Practice: The average life span of Canadian women has grown logarithmically over the past century. Rather than make a log equation to start, we are going to make an exponential equation for the inverse relationship.

Find an expression $Y(\ell)$ that gives you the year Y when the life expectancy is ℓ if the following are true

- $Y(70) = 1940$
- $Y(84) = 2010$
- The horizontal asymptote is 1900



Re-write your equation in base e.

Determine the logarithmic equation $L(y)$ that determines the life expectancy at year y

Practice: As temperature increases the amount of sugar that can be dissolved in 100mL of water increases exponentially (the solubility changes). At $20^{\circ}C$ you can dissolve 200g and every $1^{\circ}C$ the temperature increases we can dissolve 1.15% more.

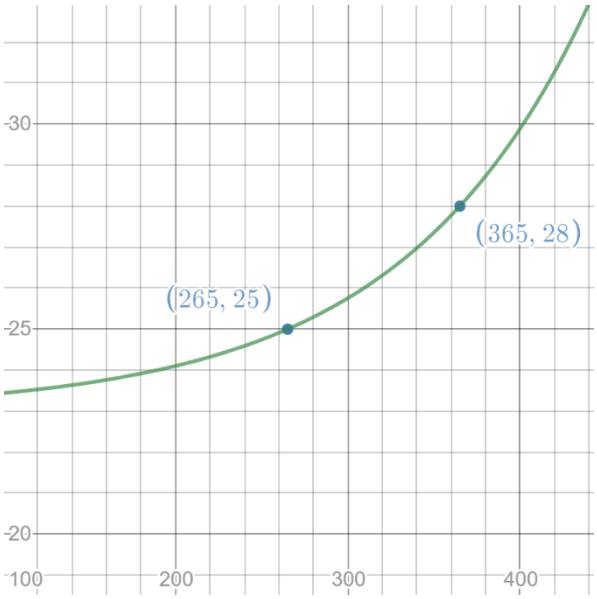
Determine an exponential equation using *e* to model how much is dissolved at a given temperature.
Determine how much is dissolved at $50^{\circ}C$

Determine a natural logarithm equation for the temperature of the water given the amount dissolved.
Determine what temperature is needed to dissolve 400g of sugar.

Practice: Athletic performance typically will follow a logarithmic growth where early growth occurs quickly and it becomes harder to progress as you improve.

Again, to determine the log equation we will consider the inverse relationship of $A(w)$ that determine the person's age given the maximum they can deadlift (a type of exercise). Consider the following

- $A(265) = 25$
- $A(365) = 28$
- The horizontal asymptote is 23



Re-write your equation in base e.

Determine the logarithmic equation $W(a)$ that determines the max weight lifted at age a.

Suggested Practice Problems: 8.4 page 412 – 415 # 1, 2, 5, 8-12, 15-17, 21
Textbook Reading: 8.4 page 404-411 Key Ideas on page 412
Next Class: Review