Solving Exponents and Logarithms Algebraically

Goal:
- Can solve equations involving logs using algebra and remember to respect the domain of log functions.
- Can model problems using the natural log thinking of the relationship to the exponential function

Terminology:
- Extraneous Solutions

When solving log equations, we need to remember to respect the domain of the original expression

$x > 0 \quad (x+1)^2 > 0 \Rightarrow x+1 \neq 0, \; x \neq -1$

Example: Solve for x

$$-3 + \log_2 x = -\log_4 (x+1)^2$$

$-3 + \log_2 x = -\frac{\log_2 (x+1)^2}{\log_2 2^2} = -\frac{1}{2}\log_2 (x+1)^2$

$+3 = +\log_2 x + \log_2 (x+1)^1$

$2^3 = 2^{\log_2 (x(x+1))}$

$8 = x^2 + x \rightarrow 0 = x^2 + x - 8$

$x = \frac{-1 \pm \sqrt{1+32}}{2}$

$= 2.37\ldots, \; -3.37\ldots$

Extraneous out of domain.

$-3 < x \quad x > 4 \quad x > -3$

Practice: Solve for x

$$\log_3 ((x+3)(x-4)) = 6 \log_{27} (x+3) + 1$$

$\log_3 ((x+3)(x-4)) = 6 \frac{\log_3 (x+3)}{\log_3 3^3} + 1$

$\log_3 (x+3)(x-4) = 2 \log_3 (x+3) + 1$

$\log_3 (x+3)(x-4) - \log_3 (x+3)^2 = 1$

$3^{\log_3 \left(\frac{(x+3)(x-4)}{(x+3)^2}\right)} = 3^1 \Rightarrow \frac{x-4}{x+3} = 3$

$x - 4 = 3x + 9$

$2x = -13$

$x = -6.5$ extraneous

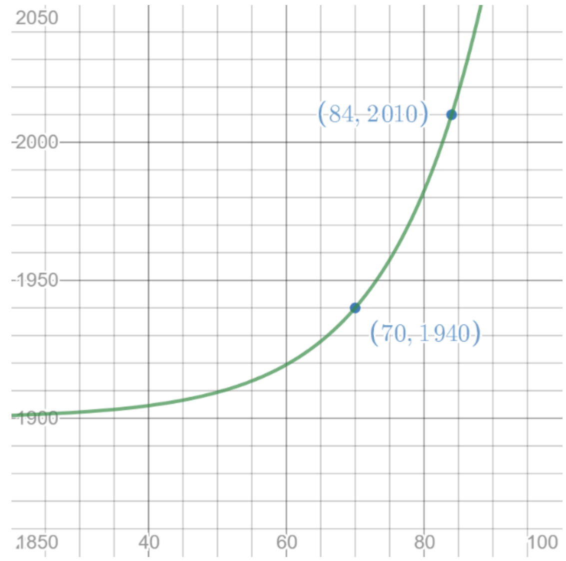
We want to practice modelling exponential and log equations and solving by changing our bases to 10 and e.

Practice: The population of the Greater Vancouver Area last year (2019) was 2.5 million. It is expected to grow at an annual rate of 0.95% for the next few decades. Write a function for the population at year t using e as the base.

Write the equation to the inverse that outputs the year given the current population.

Determine the year the population of Vancouver will reach 4 million. Determine the year when then population of Vancouver was 1 million.

Practice: The average life span of Canadian women has grown logarithmically over the past century. Rather than make a log equation to start, we are going to make an exponential equation for the inverse relationship.



Find an expression $Y(\ell)$ that gives you the year Y when the life expectancy is ℓ if the following are true

- $Y(70) = 1940$
- $Y(84) = 2010$
- The horizontal asymptote is 1900

Re-write your equation in base e.

Determine the logarithmic equation $L(y)$ that determines the life expectancy at year y

Practice: As temperature increases the amount of sugar that can be dissolved in 100mL of water increases exponentially (the solubility changes). At 20°C you can dissolve 200g and every 1°C the temperature increases we can dissolve 1.15% more.

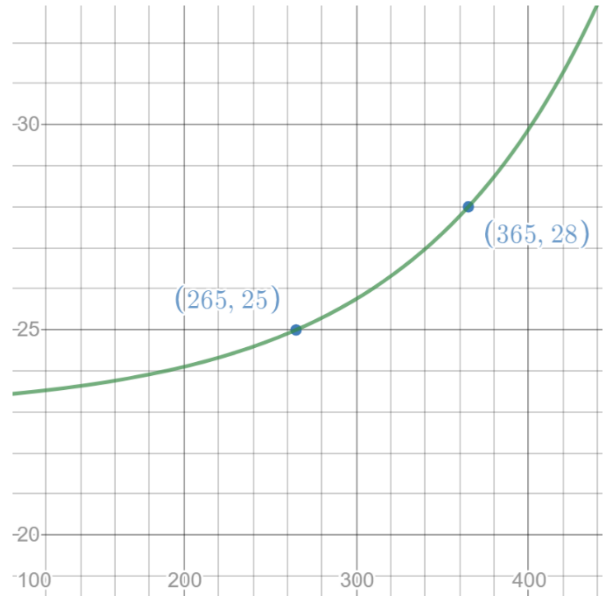
Determine an exponential equation using e to model how much is dissolved at a given temperature. Determine how much is dissolved at 50°C

Determine a natural logarithm equation for the temperature of the water given the amount dissolved. Determine what temperature is needed to dissolve 400g of sugar.

Practice: Athletic performance typically will follow a logarithmic growth where early growth occurs quickly and it becomes harder to progress as you improve.

Again, to determine the log equation we will consider the inverse relationship of $A(w)$ that determine the person's age given the maximum they can deadlift (a type of exercise). Consider the following

- $A(265) = 25$
- $A(365) = 28$
- The horizontal asymptote is 23



Re-write your equation in base e.

Determine the logarithmic equation $W(a)$ that determines the max weight lifted at age a.

Suggested Practice Problems: 8.4 page 412 – 415 # 1, 2, 5, 8-12, 15-17, 21	
Textbook Reading: 8.4 page 404-411	
Key Ideas on page 412	
Next Class: Review	