Solving Exponents and Logarithms Algebraically

Goal:
- Can solve equations involving logs using algebra and remember to respect the domain of log functions.
- Can model problems using the natural log thinking of the relationship to the exponential function

Terminology:
- Extraneous Solutions

When solving log equations, we need to remember to respect the domain of the original expression

Example: Solve for x

$$-3+\log_2 x=-\log_4(x+1)^2$$

Practice: Solve for x

$$\log_3\big((x+3)(x-4)\big)=6\log_{27}(x+3)+1$$

We want to practice modelling exponential and log equations and solving by changing our bases to 10 and e.

Practice: The population of the Greater Vancouver Area last year (2019) was 2.5 million. It is expected to grow at an annual rate of 0.95% for the next few decades. Write a function for the population at year t using e as the base.

$r = 1.0095 \quad T = 1 \quad$ Base $f(t) = 1.0095^t$

$$P(t) = 2.5\left(1.0095^{t-2019}\right)$$

$$= 2.5\, e^{k(t-2019)}$$

$1.0095 = e^k$

$k = \ln 1.0095$

Write the equation to the inverse that outputs the year given the current population.

solve for t

$$P = 2.5\, e^{k(t-2019)}$$

$$\frac{P}{2.5} = e^{k(t-2019)}$$

$$\ln\left(\frac{P}{2.5}\right) = k(t-2019)$$

$$\Rightarrow t(P) = \frac{1}{k}\ln\left(\frac{P}{2.5}\right) + 2019$$

Determine the year the population of Vancouver will reach 4 million. Determine the year when then population of Vancouver was 1 million.

$k = \ln(1.0095)$

$$t(4) = \frac{1}{k}\ln\left(\frac{4}{2.5}\right) + 2019$$

$$= 2069$$

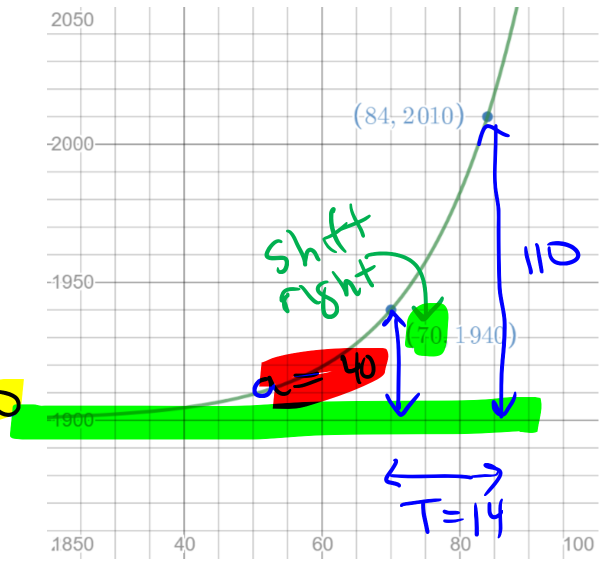
$$t(1) = \frac{1}{k}\ln\left(\frac{1}{2.5}\right) + 2019$$

$$= 1922$$

Practice: The average life span of Canadian women has grown logarithmically over the past century. Rather than make a log equation to start, we are going to make an exponential equation for the inverse relationship.

Find an expression $Y(\ell)$ that gives you the year Y when the life expectancy is ℓ if the following are true

- $Y(70) = 1940$
- $Y(84) = 2010$
- The horizontal asymptote is 1900



$T = 14$ $\qquad 40 \xrightarrow[\times r]{} 110$ $\qquad r = \frac{11}{4}$

Basic: $\left(\frac{11}{4}\right)^{\frac{t}{14}}$

$d = 1900$

Total $Y(\ell) = 40\left(\frac{11}{4}\right)^{\frac{\ell - 70}{14}} + 1900$

Re-write your equation in base e.

$$\frac{11}{4} = e^{k} \Rightarrow k = \ln\left(\frac{11}{4}\right)$$

$$Y(\ell) = 40\, e^{\frac{k}{14}(\ell - 70)} + 1900$$

Determine the logarithmic equation $L(y)$ that determines the life expectancy at year y

solve for ℓ

$$\frac{y - 1900}{40} = e^{\frac{k}{14}(\ell - 70)}$$

$$\ln\left(\frac{y - 1900}{40}\right) = \frac{k}{14}(\ell - 70)$$

$$\Rightarrow L(y) = \frac{14}{k}\ln\left(\frac{y - 1900}{40}\right) + 70$$

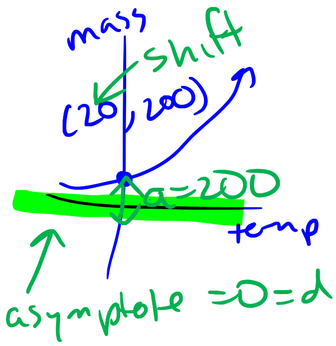
can determine life expectancy in 2060

$$L(2060) = \frac{14}{k}\ln\left(\frac{2060 - 1900}{40}\right) + 70$$

$$= 89.2 \text{ years}$$

Practice: As temperature increases the amount of sugar that can be dissolved in 100mL of water increases exponentially (the solubility changes). At 20°C you can dissolve 200g and every 1°C the temperature increases we can dissolve 1.15% more.

Determine an exponential equation using e to model how much is dissolved at a given temperature. Determine how much is dissolved at 50°C



$r = 1.0115$ $T = 1\,°C$

Basic : $(1.0115)^t$, t is temp °C

$$m(t) = 200\,(1.0115)^{t-20} = 200\,e^{k(t-20)}, \quad k = \ln 1.0115$$

$$m(50) = 200\left(e^{k(50-20)}\right) = 281.8\text{ g}$$

Determine a natural logarithm equation for the temperature of the water given the amount dissolved. Determine what temperature is needed to dissolve 400g of sugar.

solve for t

$$m = 200\left(e^{k(t-20)}\right)$$

$$\frac{m}{200} = e^{k(t-20)}$$

$$\ln\left(\frac{m}{200}\right) = k(t-20)$$

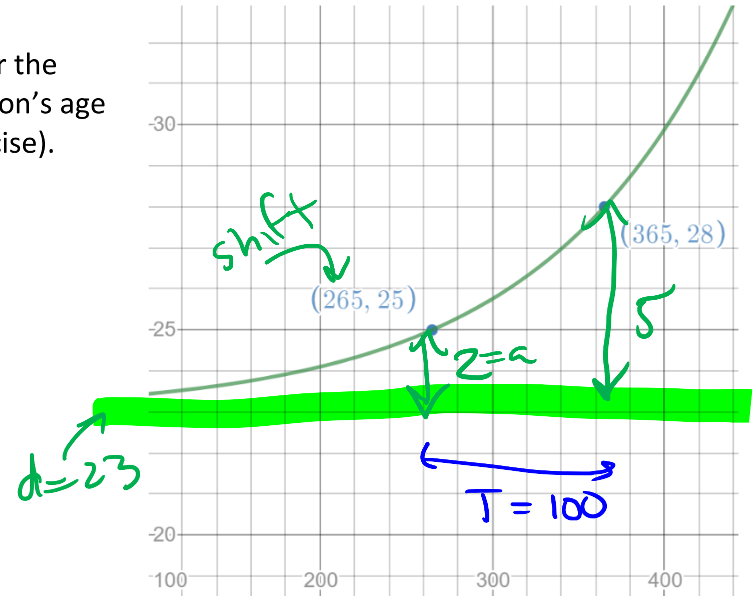
$$\Rightarrow t(m) = \frac{1}{k}\ln\left(\frac{m}{200}\right) + 20$$

$$t(400) = \frac{1}{k}\ln\left(\frac{400}{200}\right) + 20 = 80.6\,°C$$

Practice: Athletic performance typically will follow a logarithmic growth where early growth occurs quickly and it becomes harder to progress as you improve.

Again, to determine the log equation we will consider the inverse relationship of $A(w)$ that determine the person's age given the maximum they can deadlift (a type of exercise). Consider the following

- $A(265) = 25$
- $A(365) = 28$
- The horizontal asymptote is 23



T = 100 lbs $\quad 2 \xrightarrow[\times r]{} 5 \quad r = \frac{5}{2}$

Basic: $\left(\frac{5}{2}\right)^{\frac{w}{100}}$

$$A(w) = 2\left(\frac{5}{2}\right)^{\frac{w-265}{100}} + 23$$

Re-write your equation in base e.

$$A(w) = 2 \cdot e^{\frac{k}{100}(w-265)} + 23 \quad , \quad k = \ln 2.5$$

Determine the logarithmic equation $W(a)$ that determines the max weight lifted at age a.

solve for w

$$a = 2 \cdot e^{\frac{k}{100}(w-265)} + 23$$

$$\frac{a-23}{2} = e^{\frac{k}{100}(w-265)}$$

$$\ln\left(\frac{a-23}{2}\right) = \frac{k}{100}(w-265)$$

$$w(a) = \frac{100}{k}\ln\left(\frac{a-23}{2}\right) + 265$$

check $w(28) = 365$ lbs

Suggested Practice Problems: 8.4 page 412 – 415 # 1, 2, 5, 8-12, 15-17, 21	
Textbook Reading: 8.4 page 404-411	Key Ideas on page 412
Next Class: Review	