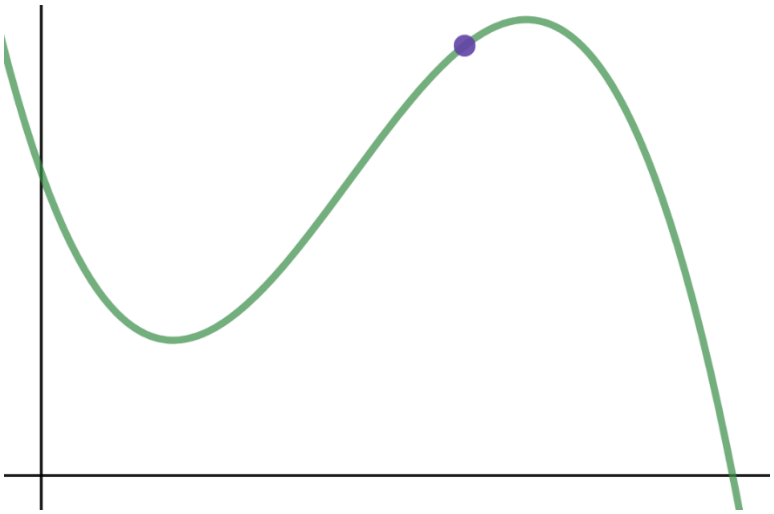
Linearization

Goal: • Understands linearization is just the tangent line at a point. • Understands that linearization "formula" is just point-slope form of tangent line.
Terminology: • Linearization

Our goal today is to approximate functions using polynomials and use our approximations to help analyze the curve.

Consider the general curve of $y = f(x)$ below



If we were to approximate $f(x)$ with a polyomial, $p(x)$, at the point $x = a$, what characteristics would you like p to have?

This is the process of **Linearization** and if we were to continue it would generate a **Taylor Series** of order n (degree of the polynomial)

$f(x) \approx \quad + \quad + \quad + \quad + \ \ldots$

Linearization (and Taylor series) are useful when we are okay with a little error around the point $x = a$.

Practice: Find the linearization of $\cos x$ around $x = \frac{\pi}{3}$

Use the linearization to estimate $\cos\left(\frac{\pi}{3} + 0.02\right)$

Is this an over-estimation or under-estimation?

Use a calculator to check the actual value of $\cos\left(\frac{\pi}{3} + 0.02\right)$

Practice: Consider the relation

$$x^2 - xy + y^3 = 1$$

Linearize the relation around the point $(1, -1)$

Use the linearization to estimate the solution to

$$1.1^2 - 1.1y + y^3 = 1$$

Are you over-estimating or under-estimating?

Practice: If $f'(x) = \frac{\sqrt{x}}{x^3+1}$ and $f(1) = 2$, estimate $f(1.1)$. Is this an over-estimation or underestimation?

Practice Problems: 4.5: # 1-14
Textbook Readings: 4.5 page 220-222
Workbook Practice: page 230-233, 238-239
Next Day: Related Rates

Related Rates (Preview)

Example: Consider a paper cone with height 10cm and radius 2cm that is full of water. A hole appears in the bottom and water begins to leave at a rate of $0.5\text{cm}^3/\text{s}$. How fast is the height of water changing after 10 seconds?

Example: In New York on New Year's Eve, a giant ball of light will drop down a flag pole as people watch. The pole is 80 feet tall and you are 100 feet from the base of the flagpole. Assume your height is 6 feet. If the ball is dropping at rate of 1.3 feet/sec, how fast is the length of your shadow growing when the light is midway down the pole?