Trig Identities

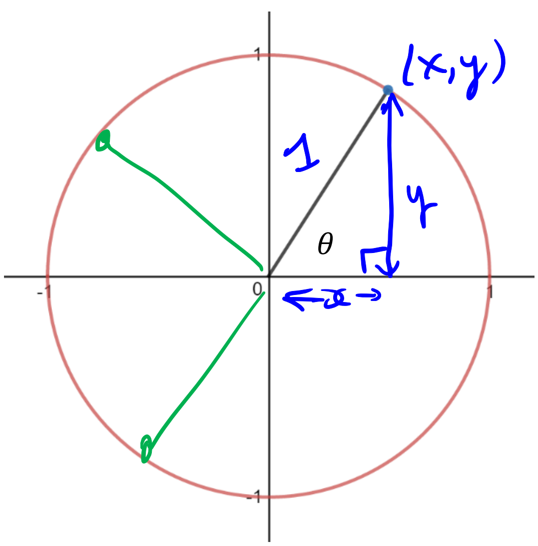
Goal:

- Can prove $\cos^2 x + \sin^2 x = 1$ using Pythagoras and can derive the other two identities from this one.
- Understands that changing all trig terms to sine and cosine is the first step to simplifying identities.
- Understands how the addition and subtraction identities are derived. Can use the identities to simplify trig statements.

New terminology:

- Statement
- Identity

Use the unit circle to show that $\sin\theta = y$ and $\cos\theta = x$.



$\cos\theta = \frac{x}{1} = x \qquad \sin\theta = \frac{y}{1} = y$

$x^2 + y^2 = 1$

$$\boxed{\cos^2\theta + \sin^2\theta = 1}$$

Pythagorean Trig Identity.

The above statement is called an **identity** – something that is fundamentally true and can be used to support other statements. In mathematics, a **statement** is something that is true or false.

$\cos^2\theta = 1 - \sin^2\theta$ $\cos\theta = \pm\sqrt{1-\sin^2\theta}$	$\sin^2\theta = 1 - \cos^2\theta$
$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$ $\boxed{1 + \tan^2\theta = \sec^2\theta}$	$\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$ $\boxed{\cot^2\theta + 1 = \csc^2\theta}$

Example: Prove or provide a counterexample to the following statements. If true, then show it.

$$\sin x = x - \frac{x^3}{6}$$

$\sin 1 = 0.841\ldots$

$1 - \frac{1}{6} = 0.833\ldots$

FALSE

$$\sec x - \sec x \sin^2 x = \cos x$$

$\frac{1}{\cos 1} - \frac{1}{\cos 1} \cdot \sin^2 1 = 0.540\ldots$

$\cos 1 = 0.540\ldots$

$\sec x - \sec x \sin^2 x$

$= \sec x - \sec x (1 - \cos^2 x)$

$= \cancel{\sec x} - \cancel{\sec x} + \frac{\cos^2 x}{\cos x}$

$= \cos x$

TRUE

Practice: Prove or provide a counterexample to the following statement. If true, then show it.

$$\csc x - \sin x = \cot x$$

$\frac{1}{\sin 1} - \sin 1 = 0.34$

$\cot 1 = 0.64$

FALSE

$$\tan^2 x \cdot \sin^2 x = \tan^2 x - \sin^2 x$$

$\tan^2 x (1 - \cos^2 x) = \tan^2 x - 1 + \cos^2 x$

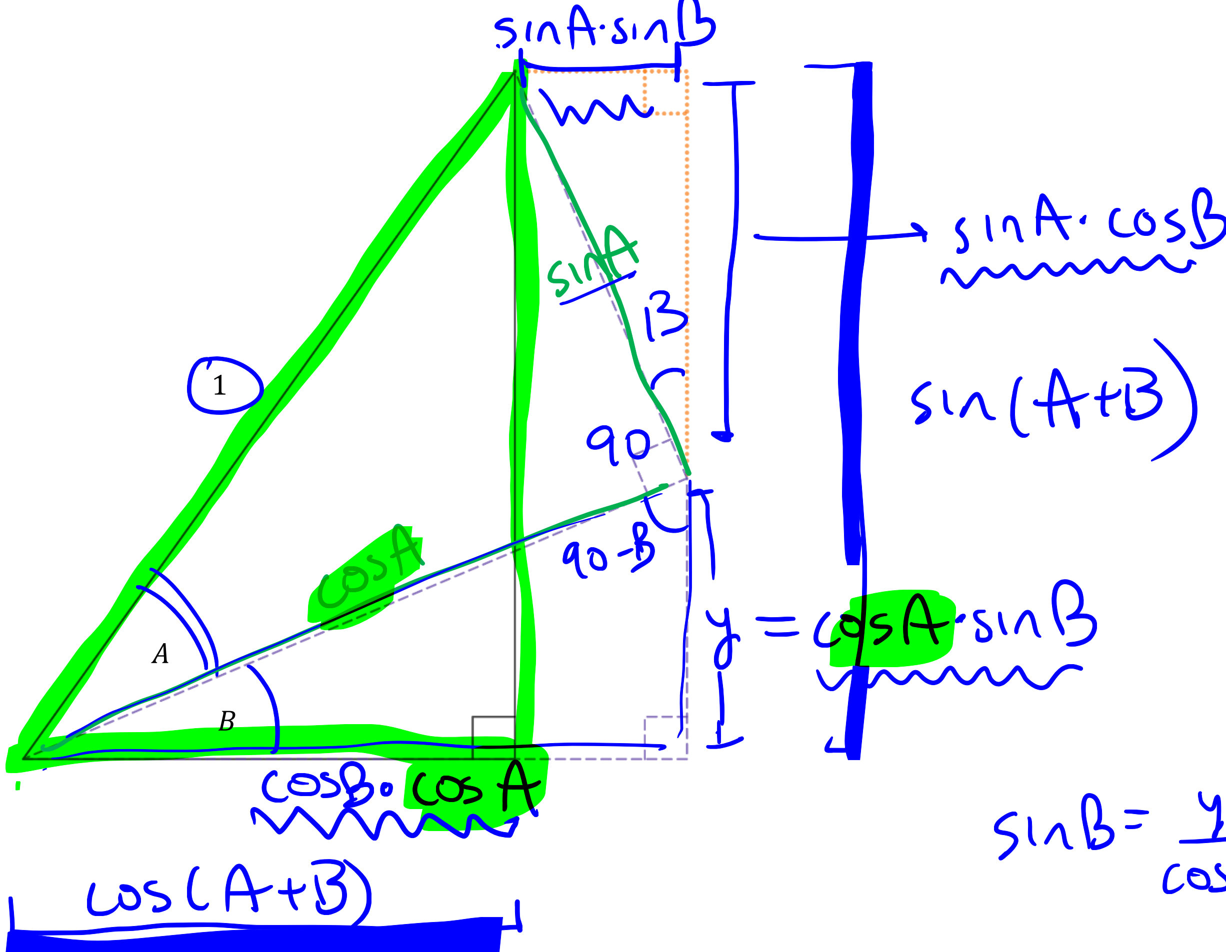
$\cancel{\tan^2 x} - \sin^2 x = \cancel{\tan^2 x} - 1 + \cos^2 x$

$-\sin^2 x = -1 + \cos^2 x$

$+\cos^2 x + \sin^2 x = +1$

True

Aside from the Pythagorean Identity, we will look at one other important identity: **Angle Addition Identity.**



$\sin B = \dfrac{y}{\cos A}$

We want to manipulate the above image so we know the following:

$\sin(A \pm B) =$ $\sin A\cos B \pm \cos A\sin B$

$\cos(A \pm B) =$ $\cos A\cos B \mp \sin A\sin B$

$\cos(-x) = \cos x$

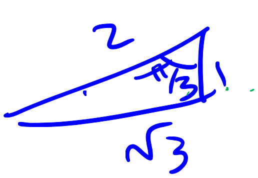
$\sin(-x) = -\sin x$

Example:

Determine the exact value of $\sin\left(\frac{\pi}{12}\right)$

$$\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$$

$$\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \sin(\pi/3)\cos(\pi/4) - \cos\left(\frac{\pi}{3}\right)\sin(\pi/4)$$



$$= \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}} - \frac{1}{2}\frac{1}{\sqrt{2}} = \boxed{\frac{\sqrt{3}-1}{2\sqrt{2}}}$$

Practice: Determine the exact value of $\cos\left(\frac{7\pi}{12}\right)$

$$\frac{7\pi}{12} = \frac{7\pi}{3} - \frac{7\pi}{4}$$

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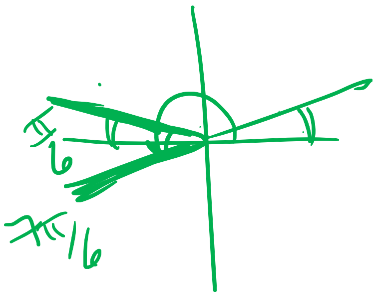
$$\cos\left(\frac{7\pi}{3} - \frac{7\pi}{4}\right) = \cos\left(\frac{7\pi}{3}\right)\cos\left(\frac{7\pi}{4}\right) + \sin\left(\frac{7\pi}{3}\right)\sin\left(\frac{7\pi}{4}\right)$$



$$= \frac{1}{2}\cdot\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\left(-\frac{1}{\sqrt{2}}\right)$$

$$= \frac{1-\sqrt{3}}{2\sqrt{2}}$$

$$\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$$

Example: Simplify the following into a single sinusoidal equation

$r\cos A\cdot\sin x + r\sin A\cdot\cos x$

Example: Determine an equation for $2\sin x - \cos x$

Practice: Determine an equation for $-3\sin x + 4\cos x$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

From the sum of angle identities, we get a set of important identities called **double angle identities**

$\sin 2A = \sin A \cos A + \cos A \sin A$

$\boxed{\sin 2A = 2\sin A \cos A}$

$\cos 2A = \cos A \cos A - \sin A \sin A$

$\boxed{\cos 2A = \cos^2 A - \sin^2 A}$

$\cos^2 A =$

$\sin^2 A =$

Suggested Practice Problems: 6.1 # 3-6, 10-12, 14-16 6.2 # 1-8, 11, 14-16, 18-20, 23, 24	
Textbook Reading: page 290-295 and 299-305 Key Ideas page 296 and 305	
Next Class: Proving Trig Identities	