Verifying Trig Identities: Part 1

Goal: • Can use Pythagorean, Sum/Difference and Double Angle identities fluently to show statements algebraically.
Terminology: • Identity

What makes the following an identity $\cos^2 x + \sin^2 x = 1$ identity

True for every $x \in \mathbb{R}$

And this just an equation $\cos^2 x - \tan^2 x = 1$ equation

True for $x = n\pi, n \in \mathbb{Z}$

Definition: An identity is different from an equation because an equation is only true for a few values, where as an identity is true for all permissible values.

The textbook describes this section as "proofs" but it really should be called good algebra. Proofs are one of the most beautiful things in mathematics and this is unfortunately not it. When you get to university, I strongly suggest you take an introductory proofs class or a class in logic and reasoning.

Example: There are at least two people in Vancouver who were born on the same minute of the same day.

There are 600K people in Vancouver.

There are $366 \times 24 \times 60 = 527\,040$ minutes

B/C 527K min < 600K people at least 2 people have the same birth minute.

Sadly, we don't get to study this ☹

$?? \; 3=3 \rightarrow \frac{3}{2}=\frac{3}{2}$

$0 \stackrel{?}{=} 0$

We do get to talk about forward and backward reasoning though.

everything is okay as long as it is reversible.

DONT multiply by 0

Example: Show the following is true:

$$\frac{(\sin x + \cos x)^2}{\cos^2 x - \sin^2 x} = \sec 2x + \tan 2x$$

$\sin^2 x, \cos^2 x$

$$\frac{\sin^2 x + 2\sin x\cos x + \cos^2 x}{\cos^2 x - \sin^2 x} = \sec 2x + \tan 2x$$

$* \cos 2x \neq 0$

$$\cos 2x \cdot \left(\frac{1+\sin 2x}{\cos 2x} = \sec 2x + \tan 2x \right)$$

$$1 + \sin 2x = 1 + \sin 2x \quad \text{done}$$

$\frac{\sin 2A}{2} = \sin A \cos A$

$\sin 2A = 2\overbrace{\sin A \cos A}^{\sin \cdot \cos}$; $\cos 2A = \overbrace{\cos^2 A - \sin^2 A}^{\cos^2 - \sin^2}$

Practice: Show the following is true

$$\sin^2 x - \sec^2 x + \tan^2 x = -\cos^2 x$$

$$\sin^2 x - (\tan^2 x + 1) + \tan^2 x = -\cos^2 x$$

$$-\sin^2 x + 1 = +\cos^2 x$$

$$\cos^2 x = \cos^2 x$$

Practice Show the following is true

$$\cot^2 x = \left(\frac{1+\sin x}{\sin x}\right)(\csc x - 1)$$

$$\cot^2 x = (\csc x + 1)(\csc x - 1)$$

$$\cot^2 x = \csc^2 x - 1$$

$$\cot^2 x = \cot^2 x$$

Practice: Show the following is true

$$\frac{\cos 2x}{\cos x + \sin x} + \sin x = \cos x$$

$$\frac{\cos^2 x - \sin^2 x}{\cos x + \sin x} + \sin x = \cos x$$

$$\frac{(\cos x - \sin x)\cancel{(\cos x + \sin x)}}{\cancel{\cos x + \sin x}} + \sin x = \cos x$$

$\cos x + \sin x \neq 0$

$$\cos x - \sin x + \sin x = \cos x$$

$$\cos x = \cos x$$

Practice Show the following is true

$$\sin^2 x + 1 = 3\sin^2 x + \cos 2x$$

$$\sin^2 x + 1 = 3\sin^2 x + 1 - 2\sin^2 x$$

$$\sin^2 x + 1 = \sin^2 x + 1$$

Practice: Show the following:

$$\tan x \cdot \sin 2x = 2 - 2\cos^2 x$$

$$\tan x \cdot 2\sin x \cos x = 2 - 2\cos^2 x$$

$\cos x \neq 0$

$$2\sin^2 x = 2 - 2\cos^2 x$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\sin^2 x = \sin^2 x$$

Practice: Show the following is true:

$$\csc x - 2\cot 2x \cdot \cos x = 2\sin x$$

$$\csc x - 2\frac{\cos 2x}{\sin 2x} \cdot \cos x = 2\sin x$$

$$\csc x - \frac{\cancel{2}(\cos^2 x - \sin^2 x)\cancel{\cos x}}{\cancel{2}\sin x \cancel{\cos x}} = 2\sin x$$

$\cos x \neq 0$

$$\frac{1}{\sin x} - \frac{\cos^2 x - \sin^2 x}{\sin x} = 2\sin x$$

$$\sin x\left(\frac{1 - \cos^2 x + \sin^2 x}{\sin x} = 2\sin x\right)$$

$\sin x \neq 0$

$$1 - \cos^2 x + \sin^2 x = 2\sin^2 x$$

$$\sin^2 x + \sin^2 x = 2\sin^2 x$$

Practice: Show the following is true:

$$(\sin x + \csc x)^2 + (\cos x + \sec x)^2 = 7 + \tan^2 x + \cot^2 x$$

$$\sin^2 x + 2\sin x \csc x + \csc^2 x + \cos^2 x + 2\cos x \sec x + \sec^2 x$$

$$= 1 + 2 + \csc^2 x + 2 + \sec^2 x$$

$$= 5 + 1 + \cot^2 x + 1 + \tan^2 x$$

$$= 7 + \cot^2 x + \tan^2 x$$

Suggested Practice Problems: 6.3 # 1-8, 10-18
Textbook Reading: page 290-295 and 309-3121; Key Ideas page 313
Next Class: Proving Trig Identities again