Verifying Trig Identities: Part 2

Example: Show the following is true for all permissible values of x.

$$\frac{1-\cos^2(2x)}{2\cos^2 x} = 2\sin^2 x$$

$$\frac{\sin^2(2x)}{2\cos^2 x} = 2\sin^2 x$$

$$\frac{(\sin 2x)^2}{2\cos^2 x} = 2\sin^2 x$$

$$\frac{(2\sin x\cos x)^2}{2\cos^2 x} = 2\sin^2 x$$

$\cos x \neq 0$

$$\frac{\overset{2}{\cancel{4}}\sin^2 x\cancel{\cos^2 x}}{\cancel{2}\cancel{\cos^2} x} = 2\sin^2 x \quad \text{done}$$

Example: Show the following is true for all permissible values

$$\frac{\csc x + 1}{\cos x} = \frac{\cot x}{1-\sin x}$$

$$\frac{(\csc x + 1)(1-\sin x)}{\cos x} = \cot x \quad , \sin x \neq 1$$

$$\frac{\csc x \cancel{+1} \cancel{-1} - \sin x}{\cos x} = \cot x$$

$$\frac{\frac{1}{\sin x} - \sin x}{\cos x} = \cot x$$

$$\frac{1-\sin^2 x}{\sin x\cos x} = \cot x$$

$\star \cos x \neq 0$

$$\frac{\cos^2 x}{\sin x \cos x} = \cot x$$

$$\frac{\cos x}{\sin x} = \cot x \quad \text{done}$$

Practice: Show the following is true for all permissible values of x

$$\frac{-\cos^3 x + \sin x \cos x - \cos x}{0.5 \sin 2x - \cos x} = \sin x + 2$$

$$\frac{-\cos x(\cos^2 x - \sin x + 1)}{\cancel{\tfrac{1}{2}} \cdot \cancel{2} \sin x \cos x - \cos x} = \sin x + 2$$

$$\frac{\cancel{-\cos x}(\cos^2 x - \sin x + 1)}{\cancel{-\cos x}(1 - \sin x)} = \sin x + 2$$

$$\frac{1 - \sin^2 x - \sin x + 1}{1 - \sin x} = \sin x + 2$$

$$\left(\frac{2 - \sin x - \sin^2 x}{1 - \sin x} = \sin x + 2\right)(1 - \sin x), \quad \sin x \neq 1$$

$$2 - \sin x - \sin^2 x = 2 - \sin x - \sin^2 x \quad \text{done}$$

Practice: Show the following is true for all permissible values of x

$$\frac{1}{1 + \sec x} - 1 = \frac{\cos x - 1}{\sin^2 x}$$

$\cos x \neq 0$

$$\frac{1}{1 + \frac{1}{\cos x}} - 1 = \frac{\cos x - 1}{\sin^2 x}$$

$$\frac{\cos x}{\cos x + 1} - 1 = \frac{\cos x - 1}{\sin^2 x}$$

$\sin x \neq 0$
$\cos x \neq -1$

$$\left(\frac{\cancel{\cos x} - \cancel{\cos x} - 1}{\cos x + 1} = \frac{\cos x - 1}{\sin^2 x}\right) \sin^2 x \cdot (\cos x + 1)$$

$$-\sin^2 x = \cos^2 x - 1$$

$$-\sin^2 x = -\sin^2 x$$

done

Practice: Show the following is true for all permissible values of x

$$\sec 2x = \frac{\csc^2 x}{\csc^2 x - 2}$$

$$\frac{1}{\cos 2x} = \frac{1}{\sin^2 x} \cdot \frac{1}{\frac{1}{\sin^2 x} - 2} \quad , \sin x \neq 0$$

$$\frac{1}{\cos 2x} = \frac{1}{1 - 2\sin^2 x}$$

$$\frac{1}{\cos 2x} = \frac{1}{\cos 2x}$$

done

Practice: Show the following is true for all permissible values of x

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

$$\frac{\sin 2x}{\cos 2x} = \frac{2\frac{\sin x}{\cos x}}{1 - \frac{\sin^2 x}{\cos^2 x}}$$

$$\frac{2\sin x\cos x}{\cos^2 x - \sin^2 x} = \frac{2\sin x}{\cancel{\cos x}} \cdot \frac{\cos^{\cancel{2}} x}{\cos^2 x - \sin^2 x} \quad , \cos x \neq 0$$

done

Practice: Show the following is true for all permissible values of x

$$4\sin^3 x = 3\sin x - \sin 3x$$

$$4\sin^3 x = 3\sin x - \sin(x+2x)$$

$$4\sin^3 x = 3\sin x - (\sin x\cos 2x + \cos x\sin 2x)$$

$$4\sin^3 x = 3\sin x - \sin x(1-2\sin^2 x) - \cos x(2\cos x\sin x)$$

$$4\sin^3 x = 3\sin x - \sin x + 2\sin^3 x - 2\sin x(1-\sin^2 x)$$

$$4\sin^3 x = 3\cancel{\sin x} - \cancel{\sin x} + 2\sin^3 x - 2\cancel{\sin x} + 2\sin^3 x$$

$$= 4\sin^3 x$$

done

Practice: Show the following is true for all permissible values of x

$$4\cos^2\left(\frac{x}{2}\right)(1+\cot^2 x) = \csc^2\left(\frac{x}{2}\right)$$

$$\cos 2x = 2\cos^2 x - 1$$

$$\cos x = 2\cos^2\left(\frac{x}{2}\right) - 1$$

&

$$\cos x = 1 - 2\sin^2\left(\frac{x}{2}\right)$$

$$4\left(\frac{\cos x + 1}{2}\right)(\csc^2 x) = \frac{1}{\sin^2\left(\frac{x}{2}\right)}$$

$$\cancel{2}\left(\frac{\cos x + 1}{\sin^2 x}\right) = \frac{1}{\frac{(1-\cos x)}{\cancel{2}}}$$

$\sin x \neq 0$

$\cos x \neq 1$

$$\left(\frac{\cos x + 1}{\sin^2 x} = \frac{1}{1-\cos x}\right)(1-\cos x)\sin^2 x$$

$$1 - \cos^2 x = \sin^2 x$$

done