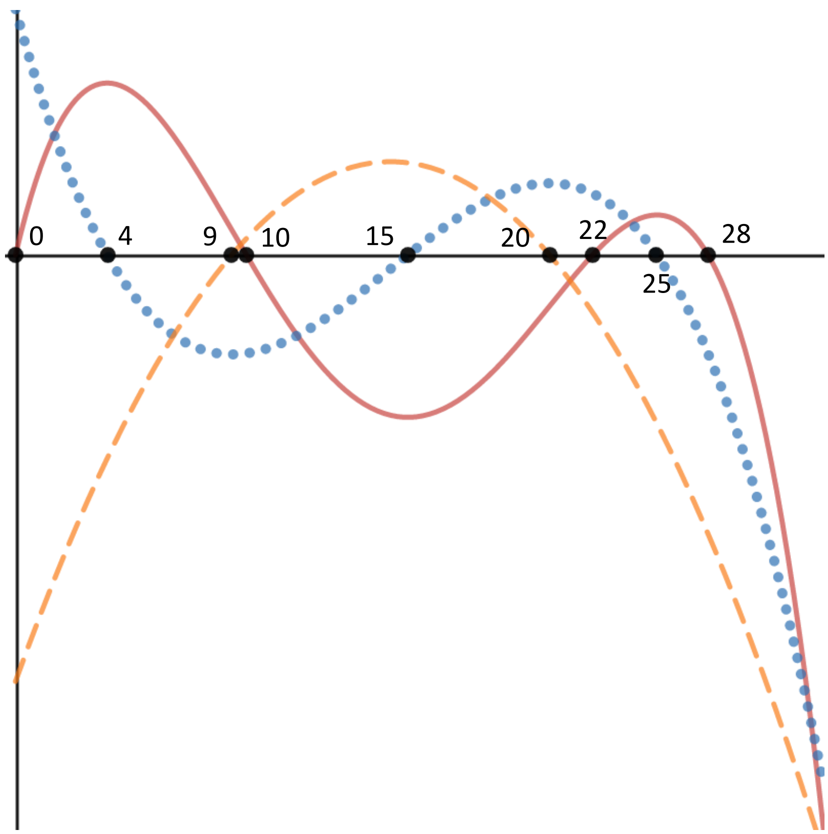
Motion and L'Hôpital's Rule

Goal:
• Can justify the use of L'Hôpital's Rule • Can identify when it's appropriate to evaluate limits using L'Hôpital's Rule • Understands the slope of a position time graph is velocity • Understands the slope of a velocity time graph is acceleration
Terminology: • Displacement • Velocity • Acceleration • L'Hôpital's Rule

Review: The solid line is the position of a particle moving in one direction (positive is to the right, negative is to the left). The dotted line is the derivative of position (velocity) and the dashed is the second derivative (acceleration).



Determine the intervals $x(t)$ is

Increasing:

Decreasing:

Concave up:

Concave down:

Example: Consider a particle moving along the x-axis (units of m) given by the equation below wher $t \in [0,6]$ in minutes

$$x(t) = 2t^3 - 21t^2 + 60t - 30$$

Determine the following:

1. Intervals the particle is moving to the right

2. Intervals the particle is moving to the left

3. Intervals the particle is speeding up

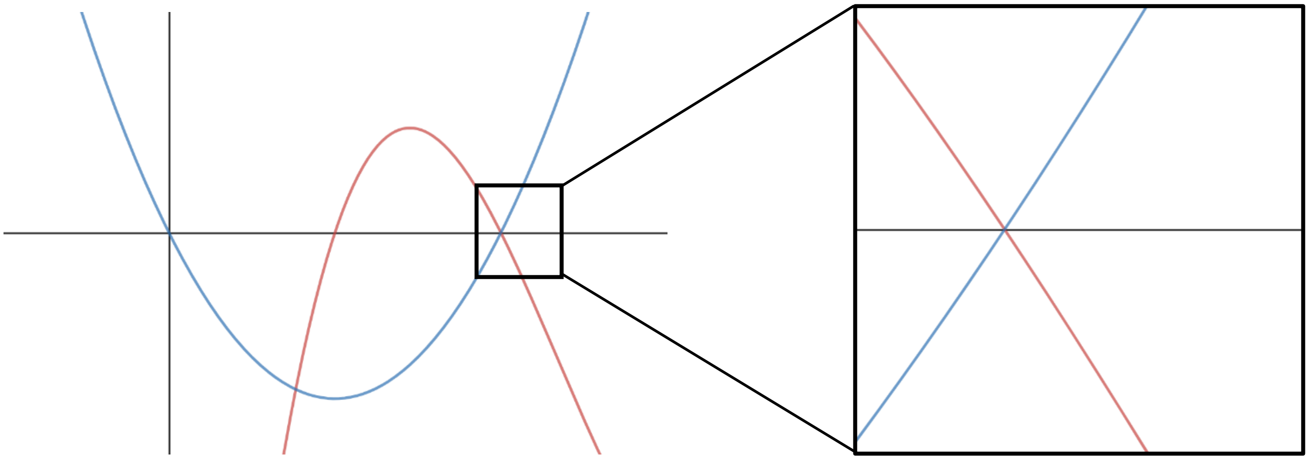
4. Intervals the particle is slowing down

5. The total displacement of the particle

We are going to finish applications of derivatives by looking back at limits and devising a new way to evaluate them. Consider the following limit:

$$\lim_{x \to c} \frac{f(x)}{g(x)}$$

Where $f(c) = g(c) = 0$. We know the limit is in an indeterminate form and believe it could have some value. Rather than looking at algebra we can look at the function near the point $x = c$ and recall that it should look like a line if it is differentiable.



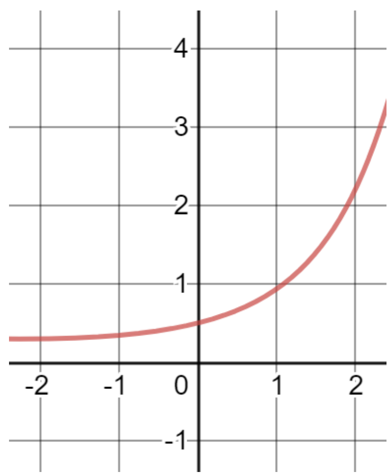
This gives us L'Hôpital's Rule:

Note that we required that:

- $f(c) = g(c) = 0$
- f and g to be differentiable around $x = c$
- The limit $\lim_{x \to c} \frac{f'(x)}{g'(x)}$ exists

Example: Evaluate the limit

$$\lim_{x \to 0} \frac{e^x - 1}{\sin x + x}$$



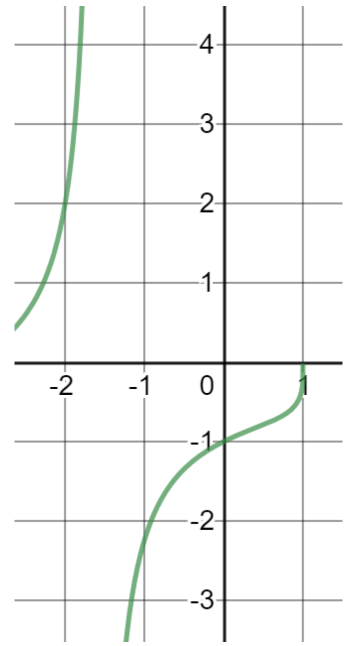
**L'Hopital also applies when you have the form $\frac{\infty}{\infty}$

Example:

$$\lim_{x \to \infty} \frac{\ln(\ln x)}{\ln x}$$

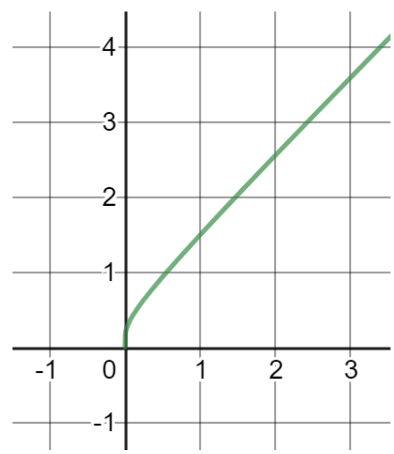
Practice: Evaluate the limit

$$\lim_{x \to 0} \frac{\tan x}{\ln(1-x)}$$



Practice: Evaluate

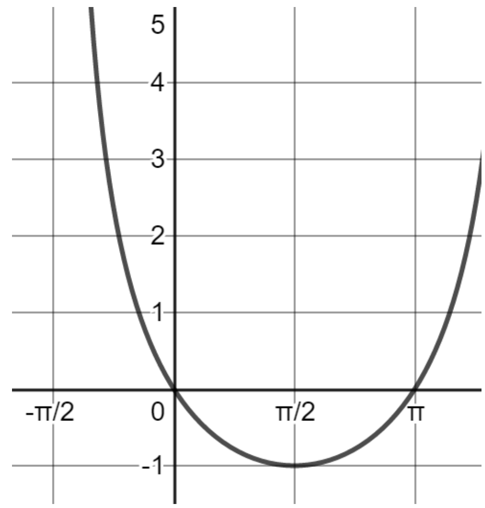
$$\lim_{x \to 1} \frac{x^2 \ln x - x + 1}{(x-1)\ln x}$$



Sometimes you will not have $\frac{0}{0}$ but instead a different form like $0 \cdot \infty$. In this case, rewrite the ∞ as the reciprocal and then you have something of the form $\frac{0}{0}$ or put 0 as the reciprocal and then you get $\frac{\infty}{\infty}$

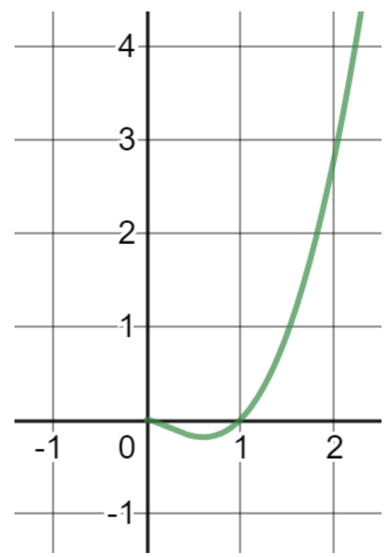
Practice: Evaluate

$$\lim_{x \to \frac{\pi}{2}} \left(x - \frac{\pi}{2}\right) \tan x$$



Practice: Evaluate

$$\lim_{x \to 0} x^2 \ln x$$



Practice Problems: 3.4 # 2-6, 12, 13 and 8.1 # 5, 6, 8, 15-24
Textbook Readings: 3.4 page 122-126 and 8.1 page 417-420
Workbook Practice: page 204-210, 240-242