Accumulation Function Practice

1. The graph of the function f shown on the right consists of a semicircle and three line segments. Let g be the function given by

$$g(x) = \int_{-3}^{x} f(t)dt - 2$$

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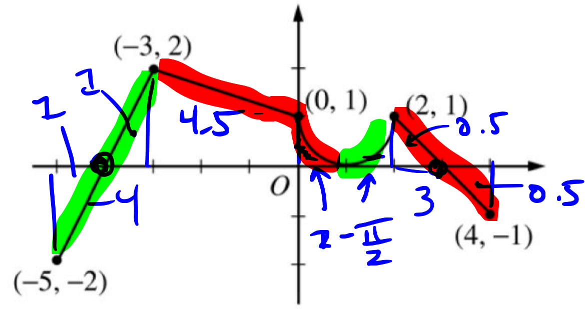


a. Find $g(0)$ and $g'(0)$.

$g(0) = \int_{-3}^{0} f\,dt - 2 = (3)(\frac{3}{2}) - 2 = 2.5$

$g'(x) = f(x) \rightarrow g'(0) = f(0) = 1$

b. Find all values of x in the open interval $(-5, 4)$ at which g attains a local extremum. Justify your answer.

$g'(x)$ changes sign when $f(x)$ changes sign

@ $x = -4$ local min (−ve to +ve)

@ $x = 3$ local max (+ve to −ve)

c. Find the absolute maximum and minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

$g(3) = \int_{-3}^{3} f\,dt - 2 = 5 + 2 - \frac{\pi}{2} - 2 \approx 3.43$ abs max

$g(-4) = \int_{-3}^{-4} f\,dt - 2 = -1 - 2 = -3$ abs min

$g(4) < g(3)$ and $g(-5) > g(-4)$

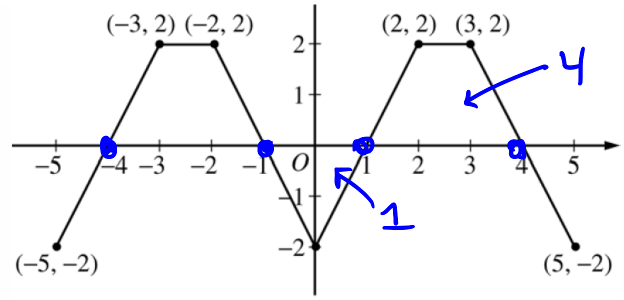
d. Find all values of x in the open interval $(-5, 4)$. At which the graph of g has a point of inflection.

$g''(x) = f'(x)$ change sign @ $x = -3, 1, 2$

e. Use technology to graph g

2. The graph of the function f shown to the right consists of six line segments.

   Let g be the function given by
   $$h(x) = \int_0^x f(t)dt$$



Graph of f

a. Find $h(4), h'(4), h''(4)$

$h(4) = \int_0^4 f\,dt = -1 + 4 = 3$

$h'(4) = f(4) = 0$

$h''(4) = f'(4) = -2$

b. Find all values of x in the open interval $(-5, 5)$ at which h attains a local extremum. Justify your answer.

$h'(x) = f(x)$ local max @ $x = -1, 4$

local min @ $x = -4, 1$

$f(x)$ changes $-$ to $+$ for min and vice versa for max

c. Suppose f is defined for all real numbers and is periodic with a period of length 5. The graph above shows two periods of f. Linearize h at $x = 108$.

$\int_a^{a+5} f(t)dt = 2$

$x \in [0,5], n \in \mathbb{Z}$

$h(x + 5n) = h(x) + 2n$

$h'(x + 5n) = h(x)$

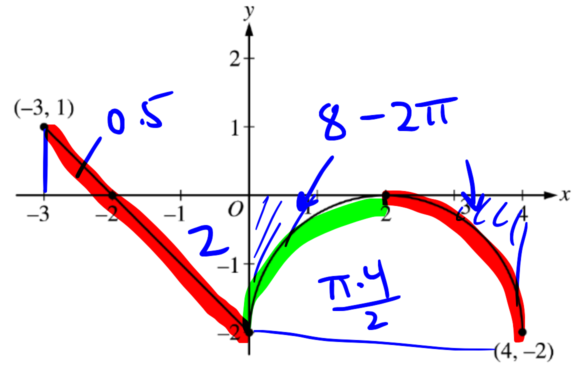
$h(108) = h(3 + 21 \cdot 5) = h(3) + 42 = 44$

$h'(108) = h'(3) = f(3) = 2$

$\Rightarrow L(x) = 2(x - 108) + 44$

3. Let f be the piecewise function defined on $[-3, 4]$ whose graph is given on the right, and let

$$k(x) = \int_0^{2x} g(t)\,dt$$



a. What is the domain of k?

$2x \in [-3, 4]$

$\Rightarrow x \in [-1.5, 2]$

b. On what interval is k increasing?

$k'(x) = g(2x)\cdot 2 > 0 \Rightarrow g(2x) > 0$

$2x \in [-3, -2] \Rightarrow x \in [-1.5, -1]$

c. Find the x values on the interval $(-3, 4)$ where k has a local extremum. Justify your answer.

$k'(x) = g(2x)\cdot 2$ change sign when $2x = -2$

@ $x = -1$

local max go +ve to −ve

d. Find the x-coordinate of each point of inflection of the graph of k on the open interval $(-3, 4)$. Justify your answer.

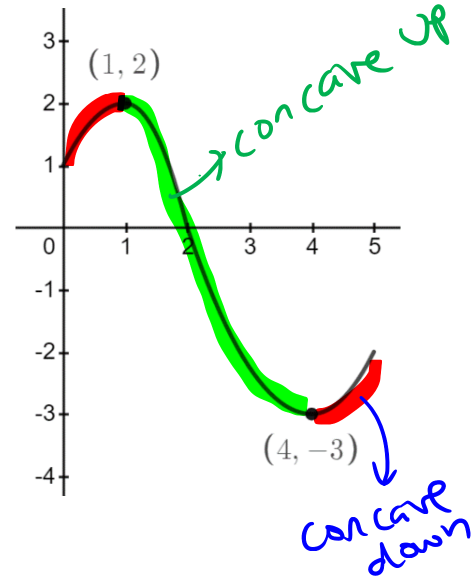
$k''(x) = 4g'(2x)$ change sign when $2x = 0, 2$

$\Rightarrow x = 0, 1$

d. Use technology to graph k

4. Let f be a function whose domain is the closed interval $[0,5]$. The graph of f is shown to the right. Let

$$m(x)=\int_{\frac{x}{2}+3}^{0} f(t)dt$$



a. What is the domain of m?

$$\frac{x}{2}+3 \in [0,5] \Rightarrow x \in [-6,4]$$

b. Find $m'(2)$

$$m'(x) = -\frac{d}{dx}\int_0^{\frac{x}{2}+3} f\,dt = -f\left(\frac{x}{2}+3\right)\cdot\frac{1}{2}$$

$$m'(2) = -\frac{1}{2}f(4) = \frac{3}{2}$$

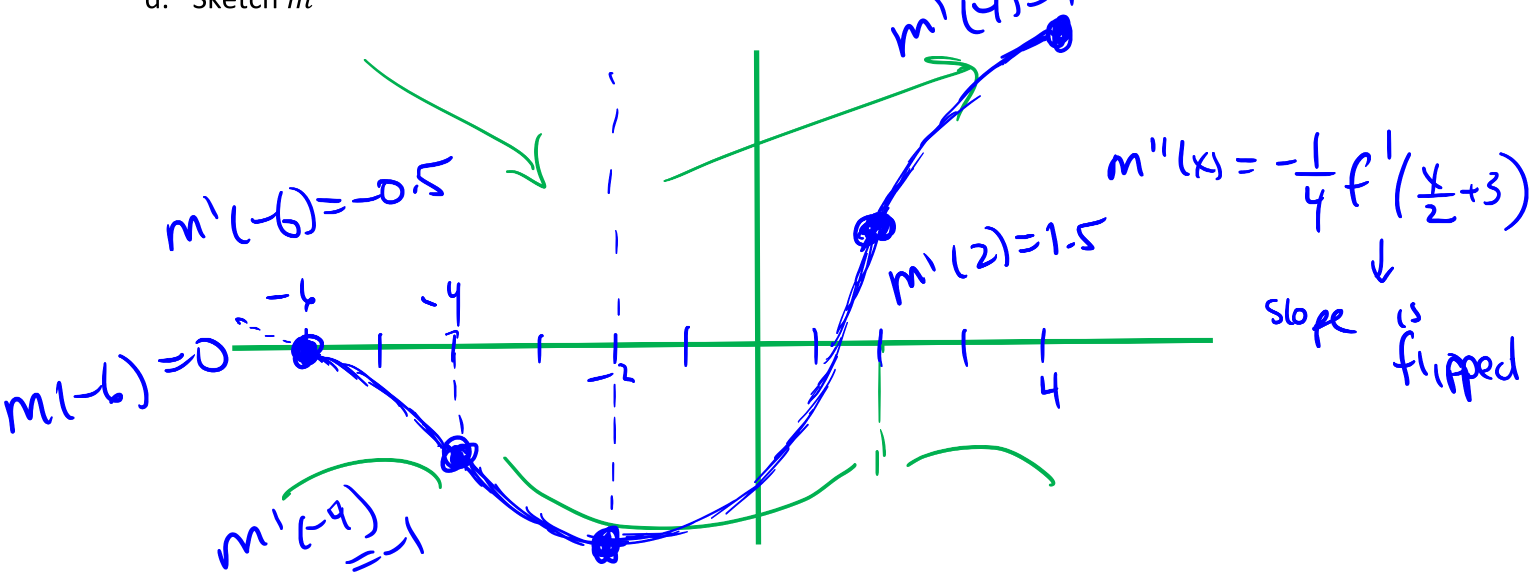
c. At what x is $m(x)$ a minimum?

$m'(x) = -\frac{1}{2}f\left(\frac{x}{2}+3\right)$ goes −ve to +ve

$\Rightarrow f\left(\frac{x}{2}+3\right)$ goes +ve to −ve

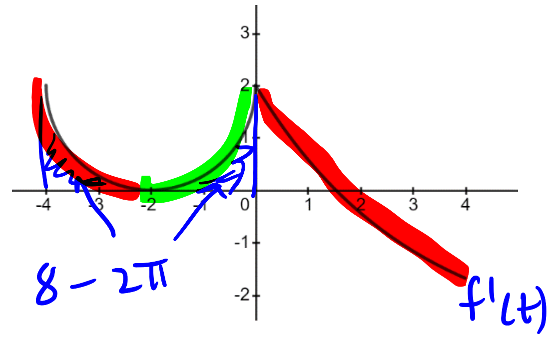
@ $\frac{x}{2}+3=2 \Rightarrow \boxed{x=-2}$

d. Sketch m



5.

$$f'(x) = \begin{cases} -\sqrt{4-(x+2)^2}+2, & -4 \le x \le 0 \\ 5e^{-\frac{x}{3}}-3, & 0 < x \le 4 \end{cases}$$



The graph of the continuous function $f'(x)$, shown in the figure above, has x-intercepts at $x = -2$ and $x = 3\ln\frac{5}{3}$. We also know that $f(0) = 5$.

a. Write an equation for $f(x)$ using an accumulation function.

$$f(x) = \int_0^x f'(t)\,dt + 5$$

b. Find $f(-4)$ and $f(4)$

$$f(-4) = \int_0^{-4} f'(t)\,dt + 5 = 2\pi - 8 + 5 \approx 3.28$$

$$f(4) = \int_0^4 f'(t)\,dt + 5 = \left(-15e^{-\frac{x}{3}} - 3x\right)_0^4 + 5$$
$$= 4.046$$

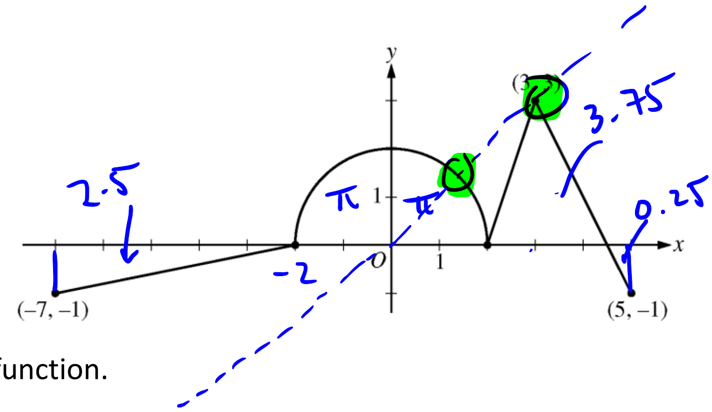
c. Find all values of $x \in (-4, 4)$ such that f has a point of inflection and a local extremum. Justify your answer.

$f'(x)$ changes sign @ $x = 3\ln 5/3$ (max)

$f''(x)$ changes sign @ $x = -2, 0$

d. Use technology to graph f.

6. The function g is defined and differentiable on the closed interval $[-7, 5]$ and satisfies $g(0) = 1$. The graph of $y = g'(x)$, the derivative of g, consists of a semi-circle and three line segments as shown on the figure to the side.



a. Write an equation for $g(x)$ using an accumulation function.

$$g(x) = \int_0^x g'(t)\,dt + 1$$

b. Find $g(-7)$ and $g(5)$

$$g(-7) = \int_0^{-7} g'(t)\,dt + 1 = -\pi + 2.5 + 1$$
$$= 0.36$$

$$g(5) = \int_0^5 g'(t)\,dt = \pi + 3.75 - 0.25$$
$$= 6.64$$

c. The function h is defined by $h(x) = g(x) - \frac{1}{2}x^2$. Find the x-coordinate of all critical points of h, where $-7 < x < 5$. Determine if they are local max/min or neither. Explain your reasoning.

$h'(x) = g'(x) - x = 0$ or $\varnothing$

$g'(x) = x \Rightarrow x = \sqrt{2}, 3$

$g'(x) - x$ doesn't change sign

$g'(x) - x$ goes +ve to −ve
$\Rightarrow$ max

d. Use technology to graph h