CURVE ANALYSIS: INFLECTION POINTS

(77-2)

1. Consider the function f defined by $f(x) = (x^2 - 1)^3$ for all real numbers x.

(a) For what values of x is the function increasing?

(b) Find the x- and y-coordinates of the relative maximum and minimum points. Justify your answer.

(c) For what values of x is the graph of f concave upward?

(d) Using the information found in parts (a), (b), and (*c*), sketch the graph of f on the axes provided.

(89BC-3)

2. Consider the function f defined by $f(x) = e^x \cos x$ with domain $[0, 2\pi]$.

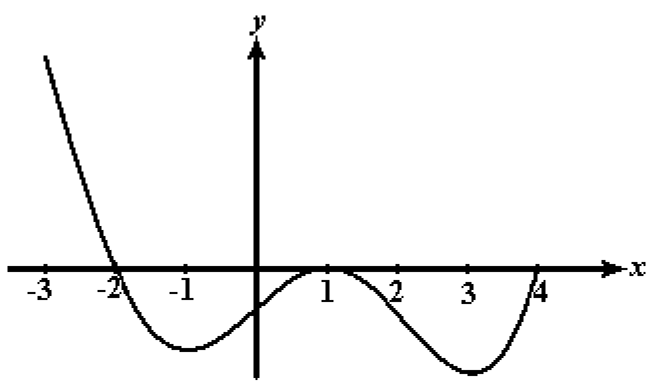
(a) Find the absolute maximum and minimum values of $f(x)$.

(b) Find the intervals on which f is increasing.

(c) Find the x-coordinate of each point of inflection of the graph of f .

(2015-5)

3.



Graph of f'

The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x-axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12 respectively.

(a) Find all x-coordinates at which f has a relative maximum. Give a reason for your answer.

(b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.

(c) Find the x-coordinates of all points of inflection for the graph of f. Give a reason for your answer.

(d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

(2008-6)

4. Let f be the function given by $f(x) = \frac{\ln x}{x}$ for all $x > 0$. The derivative of f is given by $f'(x) = \frac{1-\ln x}{x^2}$.

a) Write an equation for the line tangent to the graph of f at $x = e^2$.

b) Find the x-coordinate of the critical point of f. Determine whether this point is a relative minimum, a relative maximum or neither for the function f. Justify your answer.

c) The graph of the function f has exactly one point of inflection. Find the x-coordinate of this point.

d) Find $\lim_{x \to 0^+} f(x)$

(92BC-4)

5. Let f be a function defined by $f(x) = \begin{cases} 2x - x^2 & \text{for } x \le 1 \\ x^2 + kx + p & \text{for } x > 1 \end{cases}$

(a) For what values of k and p will f be continuous and differentiable at $x = 1$?

(b) For the values of k and p found in part (a), on what interval or intervals is f increasing?

(c) Using the values of k and p found in part (a), find all points of inflection of the graph of f. Support your conclusion.

(2001-4)

6. Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative h is given by

$$h'(x) = \frac{x^2 - 2}{x} \text{ for all } x \neq 0.$$

(a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

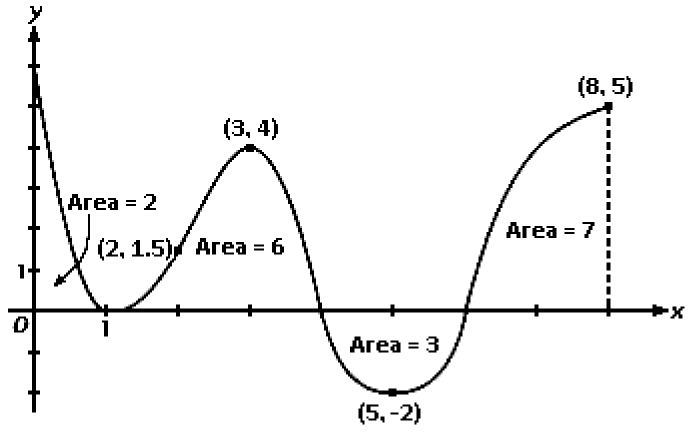
(b) On what intervals, if any, is the graph of h concave up? Justify your answer.

(c) Write an equation for the line tangent to the graph of h at $x = 4$.

(d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

(2013-4)

7.



Graph of f'

The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.

(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

(d) The function g is defined by $g(x) - (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.