MOTION

(97-1)

1. A particle moves along the *x*-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 3t^2 - 2t - 1$. The position $x(t)$ is 5 for $t = 2$.

(a) Write a polynomial expression for the position of the particle at any time $t \geq 0$.

(b) For what values of t, $0 \leq t \leq 3$, is the particle's instantaneous velocity the same as its average velocity on the closed interval [0, 3]?

(c) Find total distance travelled by the particle from time $t = 0$ until time $t = 3$.

(93-2)

2. A particle moves on the *x*-axis so that its position at any time $t > 0$ is given by $x(t) = 2te^{-t}$.

(a) Find the acceleration of the particle at $t = 0$.

(b) Find the velocity of the particle when its acceleration is 0.

(c) Find the total distance travelled by the particle from $t = 0$ to $t = 5$.

(2012-6)

3. For $0 \leq t \leq 12$, a particle moves along the *x*-axis. The velocity of the particle at time t is given by $v(t) = \cos\left(\frac{\pi t}{6}\right)$. The particle is at position $x = -2$ at time $t = 0$.

(a) For $0 \leq t \leq 12$, when is the particle moving to the left?

(b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to time $t = 6$.

(c) Find the acceleration of the particle at time *t*. Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.

(d) Find the position of the particle at time $t = 4$.

(2010(B)-6)

4. Two particles move along the *x*-axis. For $0 \leq t \leq 6$, the position of the particle P at time *t* is given by $p(t) = 2\cos\left(\frac{\pi}{4}t\right)$, while the position of particle R at time *t* is given by $r(t) = t^3 - 6t^2 + 9t + 3$.

(a) For $0 \leq t \leq 6$, find all times t during which the particle *R* is moving to the right.

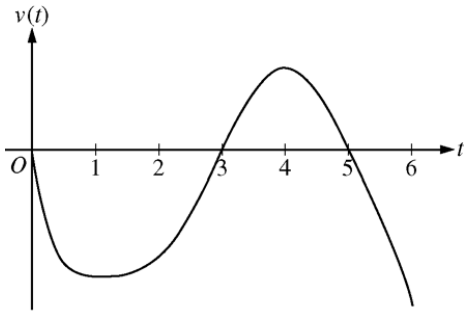
(b) For $0 \leq t \leq 6$, find all times t during which the two particles travel in opposite directions.

(c) Find the acceleration of particle P at time $t = 3$. Is particle *P* speeding up, slowing down, or doing neither at time $t = 3$? Explain your reasoning.

(d) Write, but do not evaluate, an expression for the average distance between the two particles on the interval $1 \leq t \leq 3$.

(2008-4)

5.



Graph of v

A particle moves along the x-axis so that its velocity at time t, for $0 \le t \le 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t-axis and the graph of v on the intervals $[0,3]$, $[3,5]$, and $[5,6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.

(a) For $0 \le t \le 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.

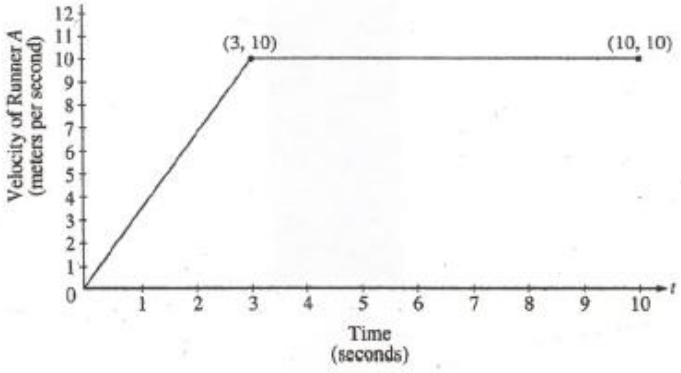
(b) For how many values of t, where $0 \le t \le 6$, is the particle at $x = -8$? Explain your reasoning.

(c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

(d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

(2000-2)

6.



Two runners, A and B, run on a straight racetrack for $0 \le t \le 10$ seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner A. The velocity, in meters per second, of Runner B is given by the function v defined by $v(t) = \dfrac{24t}{2t+3}$.

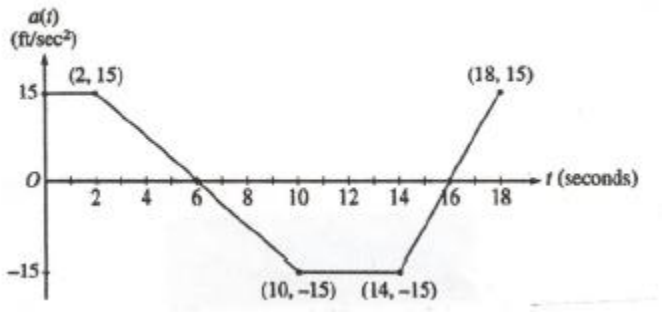
(a) Find the velocity of Runner A and the velocity of Runner B at time t = 2 seconds. Indicate units of measure.

(b) Find the acceleration of Runner A and the acceleration of Runner B at time t = 2 seconds. Indicate units of measure.

(c) Find the total distance run by Runner A and the total distance run by Runner B over the time interval $0 \le t \le 10$ seconds. Indicate units of measure.

(2001-3)

7.



A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \le t \le 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2, is the piecewise linear function defined by the graph above.

(a) Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?

(b) At what time in the interval $0 \le t \le 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?

(c) On the time interval $0 \le t \le 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.

(d) At what times in the interval $0 \le t \le 18$, if any, is the car's velocity equal to zero? Justify your answer.

(99-1)

8. A particle moves along the y-axis with velocity given by $v(t) = t\sin(t^2)$ for $t \ge 0$.

(a) In which direction (up or down) is the particle moving at time $t = 1.5$? Why?

(b) Find the acceleration of the particle at time $t = 1.5$. Is the velocity of the particle increasing at $t = 1.5$? Why or why not?

(c) Given that $y(t)$ is the position of the particle at time t and that $y(0) = 3$, find $y(2)$.

(d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

(76BC-1)

9. A particle moves on the x-axis in such a way that its position at time t is given by

$$x = (2t-1)(t-1)^2.$$

(a) At what times t is the particle at rest?

(b) During what interval of time is the particle moving to the left? Justify your answer.

(c) At what time during the interval found in (b) is the particle moving most rapidly (that is, the speed is a maximum)? Justify your answer.

(2007-4)

10. A particle moves along the x-axis with position at time t given by

$x(t) = e^{-t}\sin t$ for $0 \le t \le 2\pi$.

a) Find the time t at which the particle is furthest to the right.

b) Find the value of the constant A for which $x(t)$ satisfies the equation

$Ax''(t) + x'(t) + x(t) = 0$ for $0 < t < 2\pi$.