

RELATED RATES

(94-5)

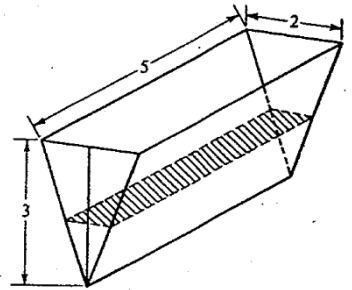
1. A circle is inscribed in a square as shown in the figure. The circumference of the circle is increasing at a constant rate of 6 inches per second. As the circle expands, the square expands to maintain the condition of tangency.



- (a) Find the rate at which the perimeter of the square is increasing. Indicate units of measure.
- (b) At the instant when the area of the circle is 25π square inches, find the rate of increase in the area enclosed between the circle and the square. Indicate units of measure.

(87-5)

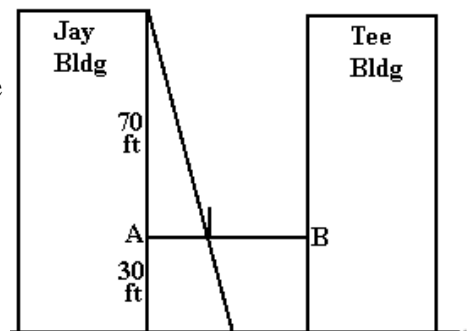
2. The trough shown in the figure above is 5 feet long, and its vertical cross sections are inverted isosceles triangles with base 2 feet and height 3 feet. Water is being siphoned out of the trough at the rate of 2 cubic feet per minute. At any time t , let h be the depth and V be the volume of water in the trough.



- (a) Find the volume of water in the trough when it is full.
- (b) What is the rate of change in h at the instant when the trough is $\frac{1}{4}$ full by volume?
- (c) What is the rate of change in the area of the surface of the water (shaded in the figure) at the instant when the trough is $\frac{1}{4}$ full by volume?

(91-6)

3. A tightrope is stretched 30 feet above the ground between the Jay and the Tee buildings, which are 50 feet apart. A tightrope walker, walking at a constant rate of 2 feet per second from point A to point B, is illuminated by a spotlight 70 feet above point A, as shown in the diagram.



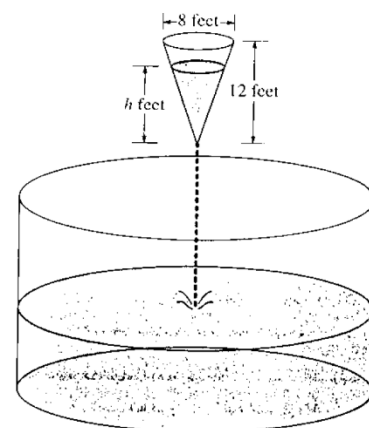
- (a) How fast is the shadow of the tightrope walker's feet moving along the ground when she is midway between the building? (Indicate units of measure.)
- (b) How far from point A is the tightrope walker when the shadow of her feet reaches the base of the Tee Building? (Indicate units of measure.)
- (c) How fast is the shadow of the tightrope walker's feet moving up the wall of the Tee building when she is 10 feet from point B? (Indicate units of measure.)

(95-5)

4. As shown in the figure, water is draining from a conical tank with height 12 feet and diameter 8 feet into a cylindrical tank that has a base with area 400π square feet. The depth h , in feet, of the water in the conical tank is changing at the rate of $(h - 12)$ feet per minute.

(The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

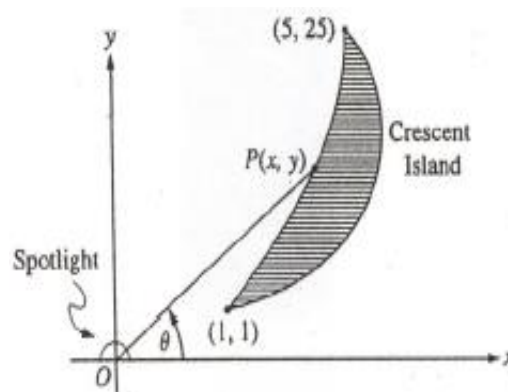
- Write an expression for the volume of water in the conical tank as a function of h .
- At what rate is the volume of water in the conical tank changing when $h = 3$? Indicate units of measure.
- Let y be the depth, in feet, of the water in the cylindrical tank. At what rate is y changing when $h = 3$? Indicate units of measure.



(96BC-6)

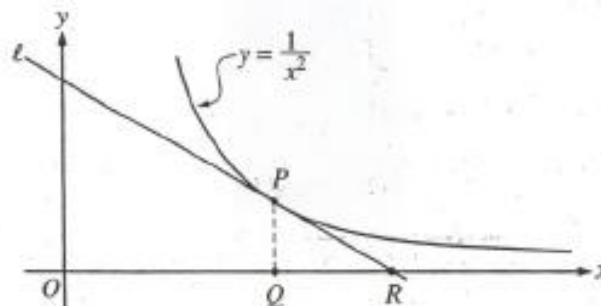
5. The figure shows a spotlight shining on point $P(x, y)$ on the shoreline of Crescent Island. The spotlight is located at the origin and is rotating. The portion of the shoreline on which the spotlight shines is in the shape of the parabola $y = x^2$ from the point $(1, 1)$ to the point $(5, 25)$. Let θ be the angle between the beam of light and the positive x -axis.

- For what values of θ between 0 and 2π does the spotlight shine on the shoreline?
- Find the x - and y -coordinates of point P in terms of $\tan \theta$.
- If the spotlight is rotating at the rate of one revolution per minute, how fast is the point P traveling along the shoreline at the instant it is at the point $(3, 9)$?



(99-6)

6.



In the figure shown, line λ is tangent to the graph of $y = \frac{1}{x^2}$ at point P , with coordinates $\left(w, \frac{1}{w^2}\right)$, where $w > 0$. Point Q has coordinates $(w, 0)$. Line λ crosses the x -axis at point R , with coordinates $(k, 0)$.

- Find the value of k when $w = 3$.
- For all $w > 0$, find k in terms of w .
- Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of k with respect to time?
- Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of the area of $\triangle PQR$ with respect to time? Determine whether the area is increasing or decreasing at this instant.