Chain Rule

Goal:
• Can build chain rule from using a series of small changes • Can apply chain rule fluently
Terminology: • Chain Rule
Reminder: • Quiz on Thursday on derivative rules 3.3-3.6

Review:

1. Show that $\frac{d}{dx}\cos x = -\sin x$

$$\lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h} = \lim_{h\to 0}\frac{\cos x\cosh - \sin x\sinh - \cos x}{h}$$

$$= \lim_{h\to 0}\cos x\frac{(\cosh - 1)}{h} - \lim_{h\to 0}\sin x\cdot\frac{\sinh}{h}$$

$$= 0 - \sin x = -\sin x$$

2. State the 4 other trig derivatives

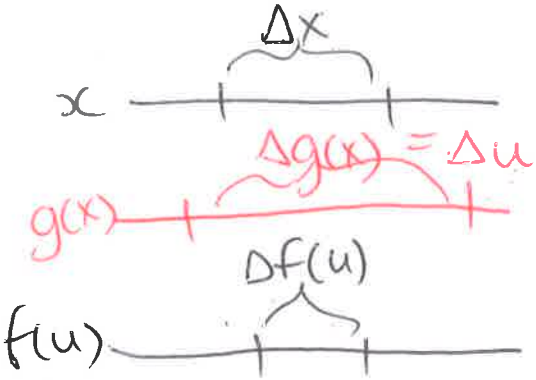
Function	Derivative
$\tan\theta$	$\left(\frac{\sin\theta}{\cos\theta}\right)' = \frac{(\sin\theta)'\cos\theta - (\cos\theta)'\sin\theta}{\cos^2\theta} = \sec^2\theta$
$\sec\theta$	$\left(\frac{1}{\cos\theta}\right)' = \frac{0+\sin\theta}{\cos^2\theta} = \frac{1}{\cos\theta}\cdot\frac{\sin\theta}{\cos\theta} = \sec\theta\tan\theta$
$\csc\theta$	$\left(\frac{1}{\sin\theta}\right)' = \frac{0-\cos\theta}{\sin^2\theta} = \frac{-1}{\sin\theta}\cdot\frac{\cos\theta}{\sin\theta} = -\csc\theta\cot\theta$
$\cot\theta$	$\left(\frac{\cos\theta}{\sin\theta}\right)' = \frac{-\sin^2\theta - \cos^2\theta}{\sin^2\theta} = \frac{-1}{\sin^2\theta} = -\csc^2\theta$

$$\text{Slope} = \frac{\Delta y}{\Delta x} \xrightarrow{\Delta x \to 0} \frac{dy}{dx}$$

We are now going to look at the derivative of function compositions, that is if $y = f(x)$ and $u = g(x)$ then what can we say about the derivative of the composition

$f(u) = f(g(x))$

$$\boxed{x} \text{———} \boxed{g(x)} \text{———} \boxed{f(g(x))}$$



$$f'(u) = \frac{\Delta f(u)}{\Delta x} = \overbrace{\frac{\Delta f(u)}{\Delta u} \cdot \frac{\Delta u}{\Delta x}}^{\text{Rational \#}} \xrightarrow{\Delta x \to 0} \underbrace{\frac{df(u)}{du} \cdot \frac{du}{dx}}_{\text{two derivatives}}$$

change wrt x

$$\frac{d}{dx}(f(g(x)) = f'(g(x)) \cdot g'(x)$$

Example: Given $f(u) = \tan u$ and $u = \sqrt{v}$ and $v = w^2 \cos w$, determine $\frac{df}{du}$, $\frac{df}{dv}$ and $\frac{df}{dw}$

$$\frac{df}{du} = \frac{df}{du} \cdot \frac{du}{du} = \sec^2 u \cdot 1 = \sec^2 u$$

$$\frac{df}{dv} = \frac{df}{du} \cdot \frac{du}{dv} = (\sec^2 u)\left(\frac{1}{2} v^{\frac{1}{2}-1}\right) = \frac{\sec^2 u}{2\sqrt{v}}$$

$$= \frac{\sec^2(\sqrt{v})}{2\sqrt{v}}$$

$$\frac{df}{dw} = \frac{df}{du} \frac{du}{dv} \cdot \frac{dv}{dw} = \frac{\sec^2(\sqrt{v})}{2\sqrt{v}} \cdot \left[2w\cos w + {}^{-}\sin w \, (w^2)\right]$$

$$= \frac{\sec^2\left(\sqrt{w^2 \cos w}\right)}{2\sqrt{w^2 \cos w}} \left(2w\cos w - w^2 \sin w\right)$$

Practice Problems: 3.6: # 1-38 (do what you need), 53, 54, 56, 57, 72

61, 67, 68

$$\frac{d^2 f}{dw^2}$$

Look Ahead: Given the relation $x^2 = y^3 - y$ what is the slope at a given point?