Derivative of a Polynomial

Goal: • Can determine the derivative of a sum and difference of functions • Can derive power rule using limits • Can use power rule to take the derivative of a polynomial • Can find higher order derivatives
Terminology: • Power Rule • Second/Third Derivative

Using the definition of the derivative, prove the following statements:

$$\frac{d}{dx}(f+g) = \frac{df}{dx} + \frac{dg}{dx}$$

$$\frac{d}{dx}(c \cdot f(x)) = c \cdot \frac{df}{dx}$$

$$\frac{d}{dx}x^n = nx^{n-1}, n \in \mathbb{N}$$

$$\frac{d}{dx}x^n = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$

$$= \lim_{h \to 0} \frac{x^n + nx^{n-1}h + \theta(h^2) - x^n}{h}$$

$$\lim_{h \to 0} nx^{n-1} + \theta(h) = nx^{n-1}$$

$(x+h)^1 = 1x + h$

$(x+h)^2 = 1x^2 + 2xh + 1h^2$

$(x+h)^3 = 1x^3 + 3x^2h + 3xh^2 + h^3$

$(x+h)^4 = 1x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$

$(x+h)^n = 1x^n + nx^{n-1}h + \theta(h^2)$

every term has at least h^2 in it

Unit 2: Rules of Differentiation

Power Rule Oct. 11

$\star\ \frac{d}{dx}x^n = nx^{n-1}$

$\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

Example: If $x = \frac{t^6}{3} + 5t^3 - 9t + 1$ then find $\frac{dx}{dt}$ ← derivative of x with respect to t

$$\frac{dx}{dt} = \frac{d}{dt}\left(\frac{t^6}{3}\right) + \frac{d}{dt}(5t^3) + \frac{d}{dt}(-9t) + \frac{d}{dt}(1)$$

$$= \frac{1}{3}\frac{d}{dt}(t^6) + 5\frac{d}{dt}(t^3) - 9\frac{d}{dt}(t^1) + 1\frac{d}{dt}(1)t^0$$

$$= \frac{1}{3}(6\cdot t^5) + 5(3t^2) - 9(1\cdot t^0) + 1(0) = 2t^5 + 15t^2 - 9$$

We are also going to introduce notation for taking the derivative repeatidly for the function $y = f(x)$

1. First Derivative:

$$y^{(1)} = y' = \frac{dy}{dx} = \frac{d}{dx}f(x)$$

slope $\Rightarrow \frac{\Delta y}{\Delta x}$ ⇒ measures the change in y as x changes

2. Second Derivative:

$$y^{(2)} \quad (y')' = y'' = \frac{d^2y}{dx^2} = \frac{d^2}{dx^2}f(x) = \frac{d}{dx}\left(\frac{d}{dx}f(x)\right)$$

↑ slope

measures the change in the slope as x changes / $\frac{\Delta y'}{\Delta x}$

3. Third Derivative

$$(y'')' = y''' = \frac{d^3y}{dx^3} = \frac{d}{dx}\left(\frac{d^2}{dx^2}f(x)\right)$$

measures the change in the change in the slope as x changes

4. Higher Order Derivatives

$$y^{(n)} = \frac{d^ny}{dx^n}$$

ie $y^{(100)}$ → 100th derivative of y

Practice Problems: 3.3: # 1-8, 27-29, 34

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