Exponential Derivatives

Goal:

- Understands that $\frac{d}{dx}e^x = e^x$
- Can use implicit differentiation to show $\frac{d}{dx}\ln x = \frac{1}{x}$

$\frac{d}{dx} x^n$

Terminology:

- None

Reminder:

- Quiz on Implicit and Logarithmic Differentiation next Wednesday
- Test on Tuesday November 12th

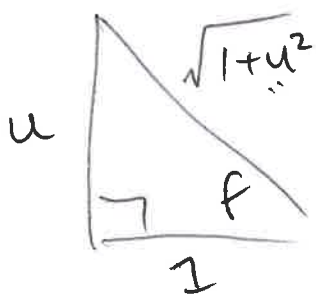
Review: Show

$$\frac{d}{dx}\underbrace{\arctan u}_{f(x)} = \frac{1}{1+u^2}\cdot\frac{du}{dx}$$

$\sqrt{x^2} = |x|$

$f = \arctan u \rightarrow \frac{d}{dx}\tan f = \frac{d}{dx}(u)$

$\sec^2 f \cdot \frac{df}{dx} = \frac{du}{dx}$

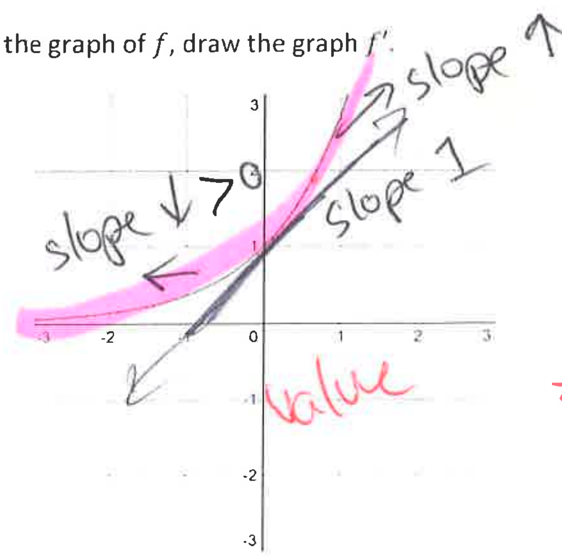


$\frac{df}{dx} = \cos^2 f \, \frac{du}{dx}$

$= \left(\frac{1}{\sqrt{1+u^2}}\right)^2 \frac{du}{dx}$

$\frac{1}{|1+u^2|}\frac{du}{dx}$

Given the graph of f, draw the graph f'.



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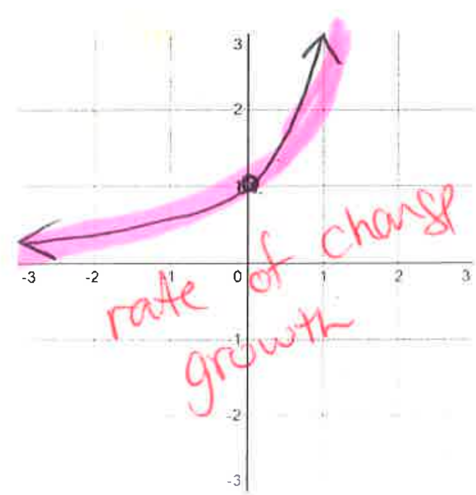

There is a very speacial number $e = 2.71828\ldots$ that is involved here. Our goal is to show that

$$\frac{d}{dx}e^x = e^x$$

And one way this is done is to use the definition of the derivative and the definition that

$$e = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$$ compound interest (continuously)

(watch the video on the website) by definition

But I want to build this function organically and give a preview of **Taylor Series** to those who are going to write the BC exam.

we want to find y such that

$y' = y$ when $x=0$ $y=1$ (to initialize the problem

$$y = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \ldots$$

same

$$y' = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \ldots$$

$\lim_{x\to\infty} y = \infty$ ✓ $b^x b^w = b^{x+w}$ if $y = b^x = y(x)$

$$b^x \cdot b^w = \left(1 + x + \frac{1}{2}x^2 + \ldots\right)\left(1 + w + \frac{1}{2}w^2 + \ldots\right)$$

$$= 1 + w + \frac{1}{2}w^2 + \ldots + x + xw + \ldots + \frac{1}{2}x^2 + \ldots$$

$$= \left(1 + (x+w) + \frac{1}{2}(x+w)^2 + \ldots\right)$$

$$= b^{x+w}$$

$$b = y(1) = \left(1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \ldots + \frac{1}{n!} + \ldots\right)$$

2! 3! 4!

$$= 2.71828\ldots = e$$

Example: Find $\frac{df}{dx}$ given that $f(u) = e^{u^2} \cdot \arccos u$

$$\star \quad \frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx}\left(\underbrace{e^{u^2}}_{F} \cdot \underbrace{\arccos u}_{G}\right)$$

$$= \underbrace{e^{u^2} \cdot 2u \cdot \frac{du}{dx}}_{F'} \underbrace{\arccos u}_{G} + \underbrace{\frac{-1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}}_{G'} \cdot \underbrace{e^{u^2}}_{F}$$

Example: And of course we want to find the derivative of the exponential inverse, logarithm. So, what is the derivative of

$$y = \ln x$$

$$y = \ln x \Rightarrow x = e^y$$

$$\frac{d}{dx} x = \frac{d}{dx} e^y$$

$$1 = e^y \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{x}$$

$$\boxed{\frac{d}{dx} \ln x = \frac{1}{x}} \quad x^{-1}$$

Example: Find $\frac{df}{dx}$ given that $f(u) = \ln(\arcsec u)$

$$\frac{df}{dx} = \frac{1}{\operatorname{arcsec} u} \cdot \frac{d}{dx}(\operatorname{arcsec} u)$$

$$= \frac{1}{\operatorname{arcsec} u} \cdot \frac{1}{|u|\sqrt{u^2-1}} \cdot \frac{du}{dx}$$

Practice Problems: 3.8: # 1-10 and 21-30 (at least every other), 41, 42   # 50, 52
Look Ahead: How can logarithms help differentiate $y = \frac{1}{x(x+1)(x+2)}$