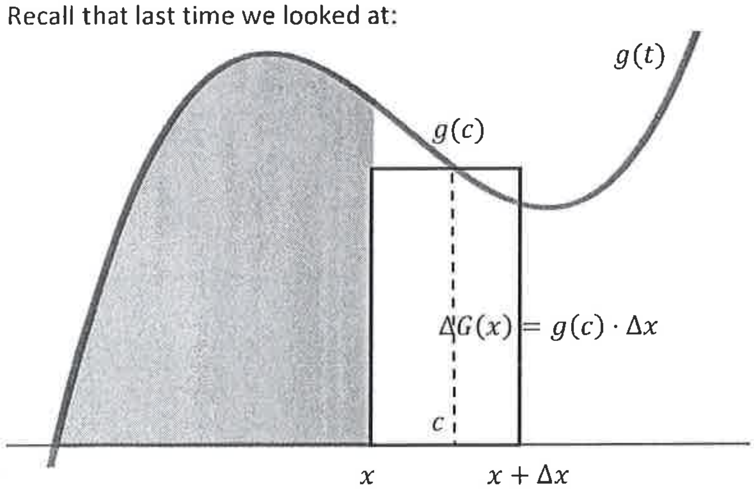
Fundamental Theorem of Calculus: Part 2

Goal: • Understands how to evaluate a definite integral for basic functions on $[a, b]$ • Understands how to derive the second part of Fundamental Theorem	
Terminology: • Total Area	
Reminder: • Test on Tuesday Feb 4	

Recall that last time we looked at:



So that $\frac{\Delta G(x)}{\Delta x} = g(c)$ and as $\Delta x \to 0$ we have that $c \to x$ so that

$$\lim_{\Delta x \to 0} \frac{\Delta G(x)}{\Delta x} = \lim_{\Delta x \to 0} g(c)$$

$$\Rightarrow \frac{dG}{dx} = g(x)$$

And we saw G is the antiderivative of g since it is some function that if we diferentiate we get $\boxed{\frac{d}{dx} G(x) = g(x)}$

So the question remains, how do we evaluate a discrete integral with antiderivates?

$$\int_a^b g(t)dt = ???$$

$$\frac{d}{dx}\boxed{\;?\;} = g(x)$$

$G(x) = \int_0^x g(t)dt$

$\Rightarrow G(x) = \int_0^x g(t)dt$

if we had $\int_0^b g(t)dt = G(b)$

if we had $\int_a^0 g(t)dt = -\int_0^a g(t)dt = -G(a)$

$\Rightarrow \int_a^b g(t)dt = \int_a^0 g(t)dt + \int_0^b g(t)dt$ → what is $G(x)$

$$\boxed{\int_a^b g(t)dt = G(b) - G(a)}$$

Example: Evaluate the following

$$\int_0^4 \underbrace{(t^2 - 4t + 1)}_{f(t)}dt = F(4) - F(0)$$

what is $F(x)$?

$$\frac{d}{dx}\underbrace{\boxed{\frac{x^3}{3} - 2x^2 + x + 2}}_{F(x)} = \underbrace{x^2 - 4x + 1}_{f(x)} \qquad F(x) = \frac{x^3}{3} - 2x^2 + x$$

$$\int_0^4 (t^2 - 4t + 1)dt = \left(\frac{4^3}{3} - 2(4)^2 + 4\right) - \left(\frac{0^3}{3} - \cancel{2(0)^2 + 0}\right)$$

$$= -6\tfrac{2}{3} = \frac{-20}{3}$$

Practice: Evaluate the following (n is a constant)

$$\int_0^1 \underbrace{(x^n + \sqrt{x})}_{f(x)}dx = I$$

$$\frac{d}{dx}\overbrace{\boxed{\frac{1}{n+1}x^{n+1} + \frac{2}{3}x^{3/2}}}^{F(x)} = x^n + \sqrt{x}$$

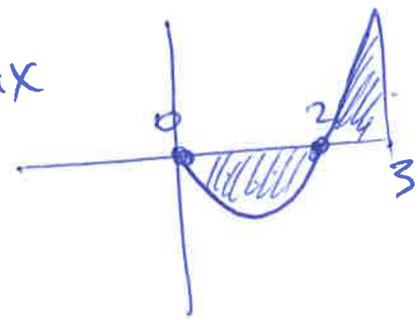
$$I = F(1) - F(0) = F(x)\Big|_0^1$$

$$= \frac{1}{n+1} + \frac{2}{3} - 0$$

$$\boxed{I = \frac{1}{n+1} + \frac{2}{3}}$$

Example: Find the **total area** between the x-axis and the curve $x^2 - 2x$ on the interval $[0, 3]$

$$\text{Area} = -\underbrace{\int_0^2 (x^2-2x)\,dx}_{\text{-ve area}} + \underbrace{\int_2^3 (x^2-2x)\,dx}_{\text{+ve area}}$$



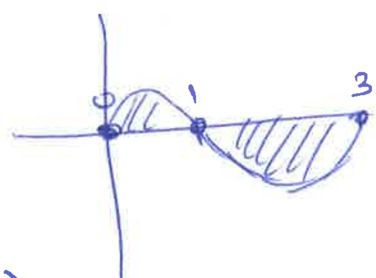
$$\frac{d}{dx}\left[\overbrace{\frac{x^3}{3} - x^2}^{F(x)}\right] = x^2 - 2x$$

$$\text{Area} = -\left[F(2) - F(0)\right] + \left[F(3) - F(2)\right]$$

$$= \underbrace{(9-\cancel{9})}_{F(3)}\ - 2\underbrace{\left(\frac{8}{3} - 4\right)}_{F(2)} + \underbrace{0}_{F(0)} = 8/3$$

Practice: Find the total are between the x-axis and the curve $x^3 - 4x^2 + 3x$ on the interval $[0, 3]$

$$\frac{d}{dx}\left[\frac{x^4}{4} - \frac{4}{3}x^3 + \frac{3}{2}x^2\right] = x^3 - 4x^2 + 3x$$



$$\text{Area} = \int_0^1 f(x)\,dx - \int_1^3 f(x)\,dx$$

$$= F(1) - F(0) - F(3) + F(1)$$

$$= 0.8\overline{33} - 0 + 2.25$$

$$= 5/6 + 9/4 = \frac{37}{12}$$

Practice Problems: 5.3 # 17-20

5.4 # 1-13,15-18, 25-28, 51

14