1.) $f(x) = \int_0^x f(t)\,dt + 5$

a.) inflection pts : where $f''(x)$ changes sign

$f''(x) = (f'(x))'$ $\Rightarrow$ slope of graph changes sign

$\uparrow$ given graph @ $x = -2, 0$

b.) $f(-4) = \int_0^{-4} f(t)\,dt + 5 = -\int_{-4}^{0} f(t)\,dt + 5$

$= (8 - 2\pi) + 5$

$= 2\pi - 3$

$f(4) = \int_0^4 f(t)\,dt + 5$

$= \int_0^4 \left(5e^{-x/3} - 3\right)dx + 5$

$=$

consider $\int_0^u \left(5e^{-x/3} - 3\right)dx = G(u)$

$\Rightarrow \frac{d}{du} G(u) = 5e^{-u/3} - 3$

$\Rightarrow$ ~~G(u)=~~ $\Rightarrow G(u) = -15e^{-u/3} - 3u$

FTC Part 2 not on Quiz

c.) on $x \in [-4, 4]$ f is maximum when $f'(x)$ goes $+$ to $-$ @ pt $5e^{-x/3} = 3$

$x = -3\ln 3/5 = 1.53...$

2. $g(x) = \int_0^x g'(t)\,dt + 5$

a) $g(3) = \int_0^3 g'(t)\,dt + 5$

$= \pi + \frac{3}{2} + 5$

$g(-2) = \int_0^{-2} g'(t)\,dt + 5$

$= -\int_{-2}^{0} g'(t)\,dt + 5$

$= -\pi + 5$

b.) inflection points of $g(x)$ when $g''(x)$ changes sign.

@ $x = 0, 2, 3$

c.) $h(x) = \int_0^x g'(t)\,dt + 5 - \frac{1}{2}x^2$

$h'(x) = g'(x) - x$ ← when does this change sign

$g'(x) - x > 0$ and then $g'(x) - x < 0$

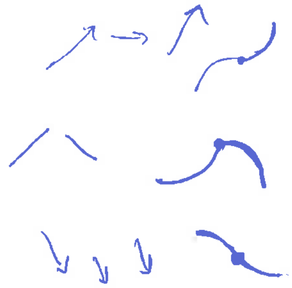
$g'(x) > \cancel{x}$ $\xrightarrow{\text{then}}$ $g'(x) < \cancel{x}$

@ $x = \sqrt{2}$

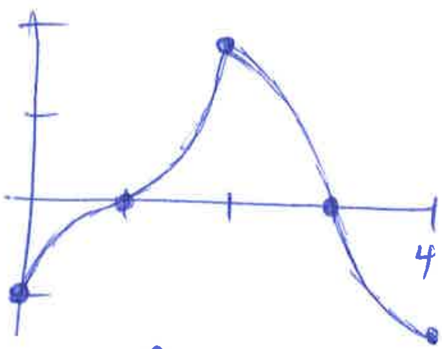
3.) a.) $x=1$

$x=2$ maximum

$x=3$



b.)



f

c.) $g(x) = \int_1^x f(t)\,dt$ find extrema of g

$g'(x) = f(x)$ go + to − OR − to +

@ $x = 1$ (min) $x = 3$ (max)

d.) $g''(x) = f'(x)$ change sign

@ $x = 2$

4.) $g(x) = \int_{-3}^{x} f(t)\,dt$

a) $g(0) = \int_{-3}^{0} f(t)\,dt = \frac{3}{2} \times 3 = \frac{9}{2}$

$g'(0) = g'(x)\big|_{x=0} = f(x)\big|_{x=0} = f(0) = 1$

b.) $x \in (-5, 4)$ g is max.

$g'(x) = f(x)$ go from + to − @ $x = 3$

c.) $x \in [-5, 4]$ g is min

$g'(x) = f(x)$ go from − to + @ $x = -4$

$g(-4) = \int_{-3}^{-4} f(t)\,dt = -\int_{-4}^{-3} f(t)\,dt$

$= -3/2$

$g(-5) = \int_{-3}^{-5} f(t)\,dt = 0 \rightarrow$ symmetry

$g(4) > 0$ so min (absolute) @ $x = -4$

d.) $x \in (-5, 4)$ g has inflection

$g''(x) = f'(x)$ go from +/− to −/+

@ $x = -3, 1, 2$

5. $f(x) = \int_0^x f'(t)\,dt + 3$

a.) when is f increasing $\Rightarrow f'(x) > 0$

on $x \in (-3, -2)$

b.) inflection pts $f''(x)$ changes sign (slope of f' changes)

@ $x = 0, 2$

c.) Linearize f @ $x=0$ & $f(0) = 3$

$L(x) = f'(0)(x-0) + f(0)$

$\boxed{L(x) = -2x + 3}$

d.) $f(-3) = \int_0^{-3} f'(t)\,dt + 3$

$= -\int_{-3}^{0} f'(t)\,dt + 3$

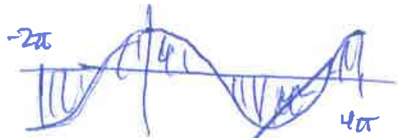
$= -\left(-\frac{3}{2}\right) + 3 = \frac{9}{2}$

$f(4) = \int_0^4 f(t)\,dt + 3$

$= -8 + 2\pi + 3$

$= 2\pi - 5$

6.) $f(x) = g(x) - \cos\frac{x}{2}$



a) $\int_{-2\pi}^{4\pi} f(x)\,dx = \int_{-2\pi}^{4\pi} g(x)\,dx - \int_{-2\pi}^{4\pi} \cos\frac{x}{2}\,dx$

$= 6\pi^2 - \left[\int_{-2\pi}^{0} \cos\frac{x}{2}\,dx + \int_{0}^{4\pi} \cos\frac{x}{2}\,dx \right]$

Let $h(x) = \int_0^x \cos\frac{t}{2}\,dt$

$\frac{d}{dx} h(x) = \cos\frac{x}{2} \rightarrow \frac{d}{dx} \underbrace{\left[2\sin\frac{x}{2}\right]}_{h(x)} = \cos\frac{x}{2}$

$\int = 6\pi^2 - \left[-h(-2\pi) + h(4\pi) \right]$

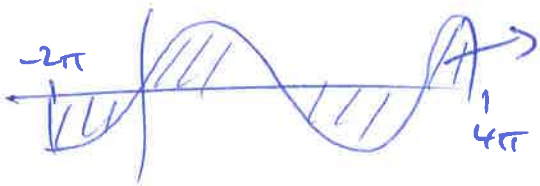
$= 6\pi^2 -$

NO COPY

6.) $f(x) = g(x) - \cos\frac{x}{2}$

a) $\int_{-2\pi}^{4\pi} f(x)\,dx = \int_{-2\pi}^{4\pi} g(x)\,dx - \int_{-2\pi}^{4\pi} \cos\frac{x}{2}\,dx$

$= 6\pi^2$

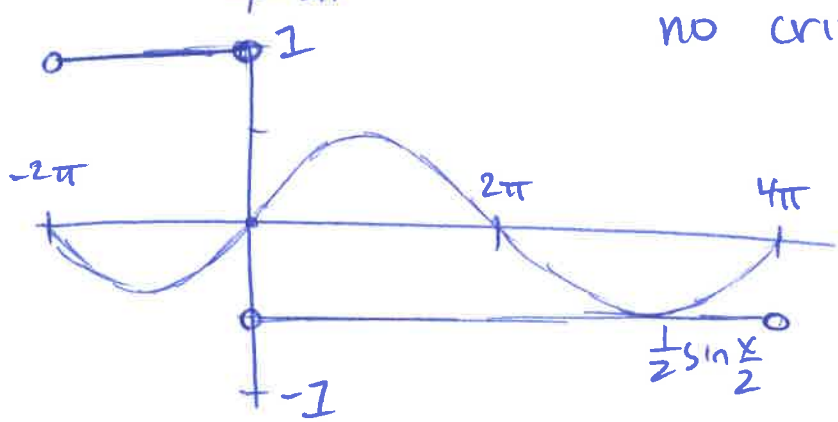


$\to 0$ using area

b.) when does f' change sign?

$f'(x) = g'(x) + \frac{1}{2}\sin\frac{x}{2}$ $\quad$ $\frac{1}{2}\sin\frac{x}{2} = -g'(x)$

no critical points!



c) $h(x) = \int_0^{3x} g(t)\,dt$

$h'(x) = 3g(3x)$

$h'(-\frac{\pi}{3}) = 3g(-\pi)$

$= 3\left(\frac{3}{2}\pi\right) = \frac{9\pi}{2}$

7.) $g(x) = \int_0^x f(t)\,dt$

a) $g(4) = \int_0^4 f(t)\,dt = \int_2^4 f(t)\,dt = 2+1 = 3$

$g'(x) = f(x) \rightarrow g'(4) = f(4) = 0$

$g''(x) = f'(x) \rightarrow g''(4) = f'(4) = -2$

b) @ $x=1$ $g'(1)$ goes from $-$ to $+$ so minimum

c) periodic length 5

$\Rightarrow g(x) = g(x+5n) \ \forall n \in \mathbb{Z}$

$g(5) = g(10) = 2$

linearize g @ $x = 108$

$L(x) = g'(108)(x-108) + g(108)$

$= g'(3)(x-108) + g(3)$

$= 2(x-108) + 2$

8.) $g(x) = \int_0^x f(t)\,dt$

a) $g(1) = \int_0^1 f(t)\,dt = 3/2$

$g'(1) = f(1) = 0$

$g''(1) = f'(1) = -3$

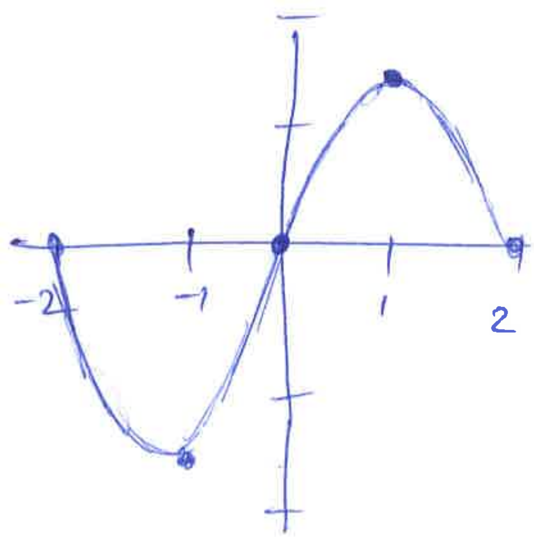
b) when is g increasing? $g'(x) > 0$

$g'(x) = f(x)$ on $(-1, 1)$

c) when is g concave down? $g''(x) < 0$

$f'(x) < 0 \Rightarrow x \in (0, 2)$

d.)



9.) $5x^3+40 = \int_c^x f(t)\,dt$

a.) $15x^2 = f(x)$

$15c^3+40 = 0 \Rightarrow c = \sqrt[3]{\frac{-\cancel{40}\,8}{\cancel{15}\,3}} = \frac{-2}{\sqrt[3]{3}}$

b.) $F(x) = \int_x^3 \sqrt{1+t^{16}}\,dt$

$F(x) = -\int_3^x \sqrt{1+t^{16}}\,dt$

$F'(x) = -\sqrt{1+x^{16}}$

10.) $h(x) = \int_0^{\frac{x}{2}+3} f(t)\,dt$ domain of f $x\in[0,5]$

$\frac{x}{2}+3 \in [0,5] \rightarrow x \in [-6,4]$ for $h(x)$

b.) $h'(2) = h'(x)\Big|_{x=2} = \frac{1}{2} f\left(\frac{x}{2}+3\right)\Big|_{x=2}$

$h'(2) = \frac{1}{2} f(4) = -3/2$

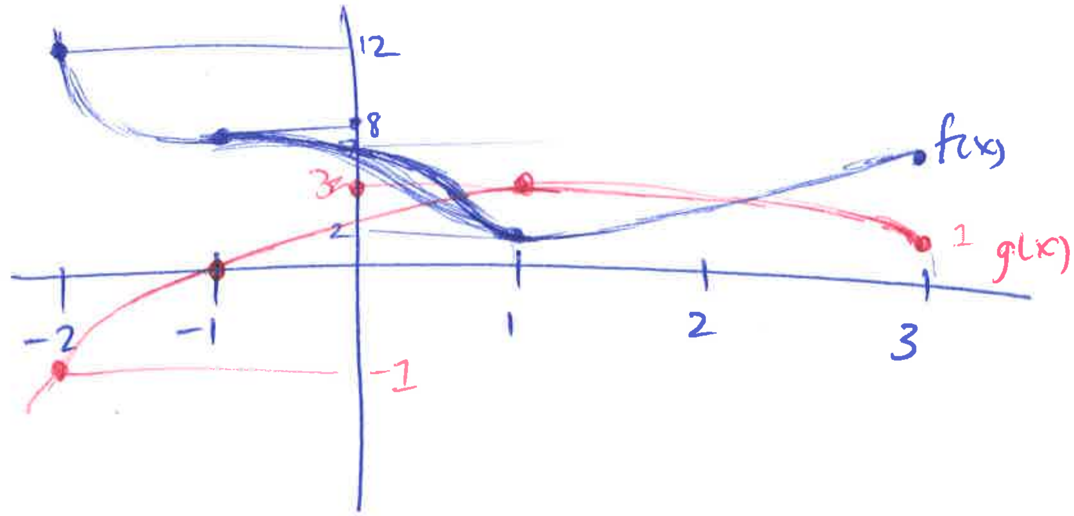
c.) h is a min when $h'(x)$ goes − to +

$h'(x) = \frac{1}{2} f\left(\frac{x}{2}+3\right)$ → has no local min (only goes + to −)

so min @ ~~x = 5 (endpoint)~~ $\frac{x}{2}+3=5$ (endpoint)

@ $x=4$

11. local min of f on [-2,3]



a) f is minimal @ $x=1$

b) $\exists$ $c \in (-1,1)$ s.t $f''(c)=0$

Since f is twice differentiable, $f''(x)$ exists $\forall$ x

$\Rightarrow$ since $f'(-1)=f'(1)=0$ and $f'(x)<0$ when $x \in (-1,1)$. As f' is continuous then by Rolle's Theorem we know $\exists$ $c \in (-1,1)$ s.t $f''(c)=0$ (f'' exists, see above)

c) $h(x) = \ln(f(x))$

$h'(x) = \frac{1}{f(x)} \cdot f'(x) \rightarrow h'(3) = \frac{f'(3)}{f(3)} = \frac{1/2}{7} = \frac{1}{14}$

d) $\int_{-2}^{3} f'(g(x))g'(x)dx \Rightarrow k(x) = \int_{a}^{x} f'(g(t))g'(t)dt$

$\Rightarrow$

$$\frac{d}{dx} k(x) = f'(g(x))g'(x)$$

$$\frac{d}{dx}\Big[\underbrace{\boxed{f(g(x))}}_{k(x)}\Big] = f'(g(x))g'(x)$$

$$f(g(x)) = \int_a^x \underbrace{f'(g(t))\,g'(t)}_{k'(t)}\,dt$$

$$\Rightarrow \int_{-2}^{3} k'(t)\,dt = \int_0^3 k'(t)\,dt + \int_{-2}^{0} k'(t)\,dt$$

$$= f(g(3)) - f(g(-2))$$

$$= f(1) - f(-1)$$

$$= 2 - 8 = -6$$

FTC Part 2

NOT on QUIZ