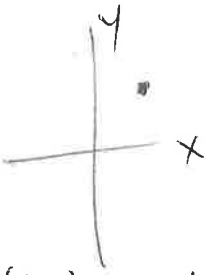
Implicit Differentiation

Goal : • Can use implicit differentiation with chain rule • Can describe the process for implicit differentiation
Terminology: • Implicit Differentiation

We are going to have a taste of calc 3 (multivariable calculus) by considering *relationships* of two or more variables rather than functions of one variable.

Consider the relation $f(x,y) = x \cdot \sin(xy)$, what does this even look like?

curvy, periodic (kinda), 3D



$x=1 \quad y=1 \quad f(1,1) = \sin 1$

For now we only want two variables and so we set $f(x,y) = 1$ (or some other constant so the relation is frozen in the x,y plane) and then ask ourselves: What is the slope of the curve at the point $(-2,\ 0.262)$?

$\frac{dy}{dx} = ?$

what is the change wrt x?

$x\sin(xy) = 1$

$\frac{d}{dx}(x \cdot \sin(xy)) = \frac{d}{dx} 1$

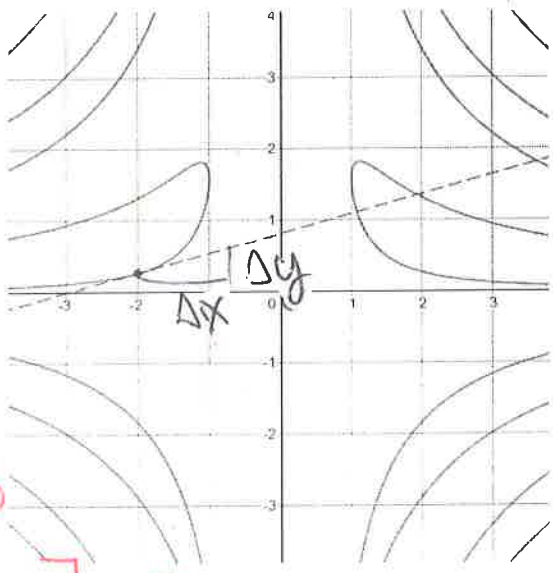
$\left(\frac{d}{dx}x\right)\sin(xy) + x\frac{d}{dx}\sin(xy) = 0$

$\sin(xy) + x\left[\cos(xy)\frac{d}{dx}(xy)\right] = 0$

$\sin(xy) + x\left[\cos(xy)\left[y\frac{dx}{dx} + x\frac{dy}{dx}\right]\right] = 0$

$-\frac{1}{2} + -2\left(\frac{\sqrt{3}}{2}\right)\left(0.262 - 2\frac{dy}{dx}\right) = 0$

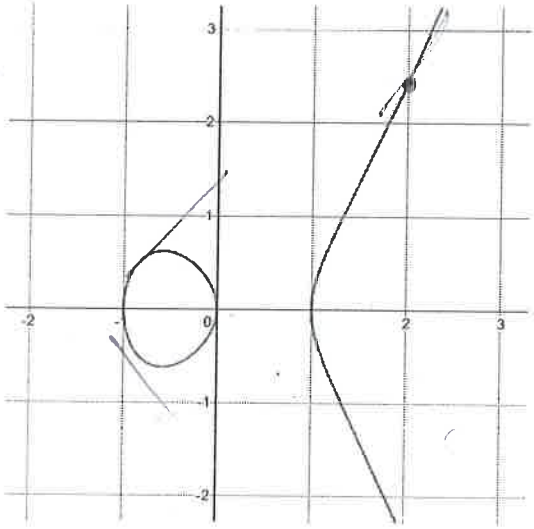
$\frac{dy}{dx} = \ldots$



$y = \pm\sqrt{x^3 - x}$

Example: Determine the slope of the curve $y^2 = x^3 - x$ at the point $(2, \sqrt{6})$

$\frac{dy}{dx} = ?$ ⇒ find change in y wrt x



$\frac{d}{dx}(y^2) = \frac{d}{dx}(x^3 - x)$

$2y\frac{dy}{dx} = 3x^2 - 1$

$\frac{dy}{dx} = \frac{3x^2 - 1}{2y}$

$= \frac{3x^2 - 1}{\text{maybe } \pm 2\sqrt{x^3 - x}}$

@ $x = 2$ $\quad \frac{dy}{dx} = \frac{11}{2\sqrt{6}}$

Example: Show that power rule works for any $n \in \mathbb{Q}$

$y = x^{a/b}$ find $\frac{dy}{dx}$

$y^b = x^a$

$\frac{d}{dx}(y^b = x^a)$

$b y^{b-1} \cdot \frac{dy}{dx} = a x^{a-1}$

$\frac{dy}{dx} = \frac{a x^{a-1}}{b y^{b-1}} \cdot \frac{y}{y} = \frac{a y x^{a-1}}{b y^b} = \frac{a x^{\frac{a}{b}} x^{a-1}}{b x^a}$

$\boxed{\frac{dy}{dx} = \frac{a}{b} x^{\frac{a}{b} - 1}}$

Practice Problems: 3.7: # 1-20 (do what you need), 27-36 (do what you need), 37a, 38, 39, 41, 46

42, 47, 50

Look Ahead: What is the derivative of $\arcsin x$?

$\frac{d}{dt}(x \sin x) = \frac{dx}{dt}\sin x + \cos x \cdot \frac{dx}{dt} x$