# Inverse Trig Derivatives

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|---|
| **Goal:**<br>• Can use implicit differentiation to find inverse function derivatives<br>• Can quickly determine the derivatives of inverse trig functions. |
| **Terminology:**<br>• Inverse Functions |
| **Reminder:**<br>• Quiz on Implicit and Logarithmic Differentiation next Wednesday<br>• Test in 2 weeks |

**Review:** Determine the slope, $dy/dx$, of the curve below at $(0,0)$

$$\tan(y^2) = y + x$$

$$\frac{d}{dx}(\tan y^2) = \frac{d}{dx}(y+x)$$

$$2y\sec^2(y^2)\frac{dy}{dx} = \frac{dy}{dx} + 1$$

$$\frac{dy}{dx} = \frac{1}{2y\sec^2(y^2) - 1} \qquad \left.\frac{dy}{dx}\right|_{y=0} = -1$$

If the the volume of a sphere is given by

$$V = \frac{4}{3}\pi r^3$$

Determine an expression that relates the rate the volume and radius are changing **with respect to time.**

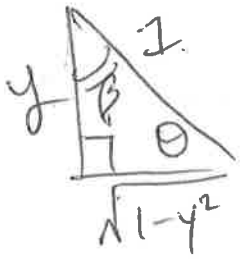
$\frac{dV}{dt}$, $\frac{dr}{dt}$ ?

$$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$$

$$\boxed{\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}}$$

If $\sin\theta = y$ then what is $\cos^{-1} y$? ⇒ angle

(angle: $\theta$; ratio: $y$; $\cos^{-1}$ y: ratio)




$$\cos^{-1}(y) = \beta = \boxed{\frac{\pi}{2} - \theta}$$

~~$\cos(\sin\theta)$~~ $\cos(\sin^{-1} y)$

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We want to expand our knowledge of trig derivatives to include their inverse functions.

| Function | Derivative | Inverse | Inverse Derivative |
|---|---|---|---|
| $\sin x$ | $\cos x$ | $\arcsin x$ | $\frac{1}{\sqrt{1-x^2}}$ |
| $\cos x$ | $-\sin x$ | $\cos^{-1} x$ | $\frac{-1}{\sqrt{1-x^2}}$ |
| $\tan x$ | $\sec^2 x$ | $\arctan x$ | $\frac{1}{1+x^2}$ |
| $\csc x$ | $-\csc x \cot x$ | $\operatorname{arccsc} x$ | $\frac{-1}{\lvert x\rvert\sqrt{x^2-1}}$ |
| $\sec x$ | $\sec x \tan x$ | $\sec^{-1}(x)$ | $\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$ |
| $\cot x$ | $-\csc^2 x$ | $\cot^{-1}(x)$ | $\frac{-1}{1+x^2}$ |

How can we differentiate

$$y = \arcsin x = \sin^{-1} x$$

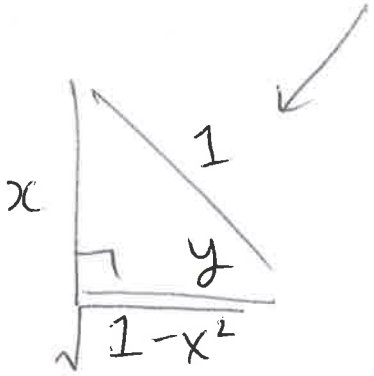
$y = \sin^{-1} x \Rightarrow x = \sin y$

$\frac{d}{dx} x = \frac{d}{dx} \sin y$

$1 = \cos y \cdot \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}$

✡ remember

$\cos y = \frac{\text{adjacent}}{\text{hypotenuse}}$




**Practice Problems**: 3.8: # 1-18 (every other), 19, 20, 27, 30, 32

# 21, 22

**Look Ahead**: If you wanted to construct a function $y = f(x)$ such that $y' = y$ what would you do?