Derivative of a Product and Quotient

Goal: • Can determine the derivative of a product and quotient of functions • Understands how limits are used to define these rules and can intuit a
Terminology: • Product Rule • Quotient Rule

Review: Determine $\frac{dA}{dt}$ given that $A = (12 + 1.5t)(156 + 13t)$.

$$A = 1872 + 156t^{1} + 234t + 19.5t^{2}$$

$$\frac{dA}{dt} = 0 + 156\cancel{t}^{1} + 234\cancel{t}^{1} + 2\cdot 19.5t^{1}$$

$$= \boxed{390 + 39t}$$

Rather than having to do the long and often tedious expansion, we ask ourselves can we determine the general derivative of a product?

$$\frac{d}{dx}(f \bullet g) = (f \bullet g)' = ???$$

$$\frac{d}{dx}(f \bullet g) = (f \bullet g)' = \lim_{h\to 0}\frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

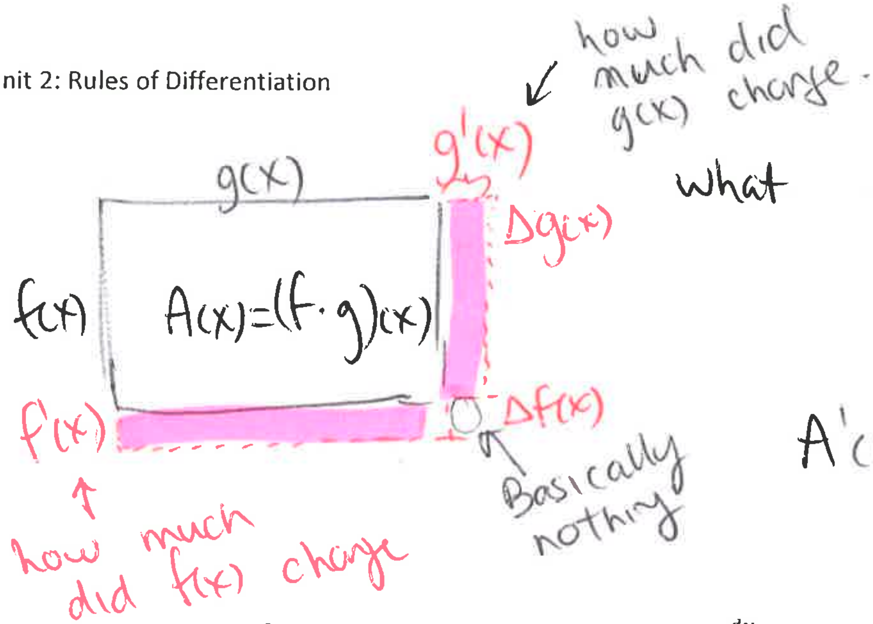
$$= \lim_{h\to 0}\frac{f(x+h)g(x+h) - f(x+h)g(x) + f(x+h)g(x) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0} f(x+h)\cdot\lim_{h\to 0}\frac{g(x+h) - g(x)}{h} + g(x)\lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

$$= g(x)\cdot\lim_{h\to 0}\frac{f(x+h) - f(x)}{h} + f(x)\cdot\lim_{h\to 0}\frac{g(x+h) - g(x)}{h}$$

$$= g\frac{df}{dx} + f\frac{dg}{dx} = f'g + g'f$$

But why would you think to do this??



what is $A'(x) = ?$

The change in Area.

$A'(x) = g'(x)f(x) + f'(x)g(x)$

Example: Given $y = \underbrace{(4x^2 - 7x + 1)}_{f(x)}\underbrace{(3x^4 + x^3 - 2x)}_{g(x)}$ determine $\frac{dy}{dx}$.

$f'(x) = 4 \cdot 2x' - 7$
$= 8x - 7$

$g'(x) = 3 \cdot 4x^3 + 3x^2 - 2$

$y' = \underbrace{(8x-7)}_{g'}\underbrace{(3x^4+x^3-2x)}_{f} + \underbrace{(12x^3+3x^2-2)}_{f'}\underbrace{(4x^2-7x+1)}_{g}$

The quotient rule has a similar style argument and is defined as

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$$

Example: Determine y' given that $y = \frac{3r^4 - 2r}{r + 4r^2}$

$f(r) = 3r^4 - 2r$

$f'(r) = 12r^3 - 2$

$g(r) = r + 4r^2$

$g'(r) = 1 + 8r$

$$y' = \frac{\overset{f'}{(12r^3-2)}\overset{g}{(r+4r^2)} - \overset{g'}{(1+8r)}\overset{f}{(3r^4-2r)}}{\underbrace{(r+4r^2)^2}_{g^2}}$$

Practice Problems: 3.3: # 9, 10, 13-19, 22-24, 33

39, 40

what is $(fgh)' = (fg)'h + fgh'$