Area Under Curve Quiz

1. Consider the function 

\[ f(x) = \sin(x^2) \]

on the interval \([0, 2]\).

(a) Using 4 subintervals, use RRAM to approximate the area under the curve. Do this by hand.

\[
\text{Area} \approx \frac{1}{2} \left( f(0.5) + f(1) + f(1.5) + f(2) \right) \\
= 0.555
\]

(b) Using 4 subintervals, use Trapezoid Method to approximate the area under the curve. Do this by hand.

\[
\text{Area} \approx \frac{1}{2} \left( \frac{1}{2} f(0) + f(0.5) + f(1) + f(1.5) + \frac{1}{2} f(2) \right) \\
= 0.744
\]

(c) Use your calculator to find the area under the curve accurate to 6 decimal places.

\[
\int_{0}^{2} \sin(x^2) \, dx = 0.8094776
\]
2. Given the graph of \( \omega \) below and that

\[
\int_0^1 \omega(x)dx = 9
\]

\[
\int_2^4 (\omega(x) - 4)dx
\]

Determine the value of

State any assumptions you made.

\[
\text{half and } \omega \text{ is symmetric about } x=2
\]

Area box is 8 \((2 \times 4)\)

\[
\int_2^4 \omega(x)dx = 3.5
\]

\[\boxed{? = 1 - 8 = -4.5}\]

3. Evaluate the following limit using geometry and symmetry

\[
\lim_{n \to \infty} \sum_{k=1}^{n} \left( \frac{4k}{n} - 1 \right) \frac{1}{n} dx
\]

\[
\frac{b-a}{n} = \frac{4}{n}
\]

Starts @ \( k=1 \) as \( n \to \infty \)

\[
\frac{4}{n} - 1 \to -1
\]

Ends @ \( k=n \) as \( n \to \infty \)

\[
\frac{4n}{n} - 1 \to 3
\]