

# Trig Derivatives

**Goal:**

- Understands the relationship between sine and cosine regarding the derivative.
- Can find the derivative of all 6 trig functions using derivative rules.

**Terminology:**

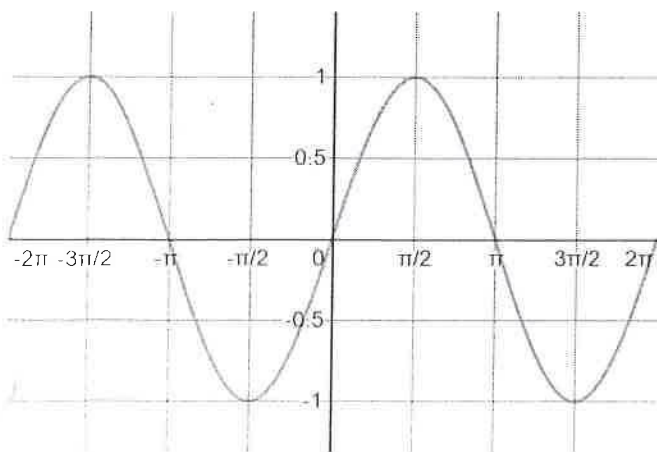
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**Reminder:**

- Quiz on Thursday on derivative rules 3.3-3.6

During our tour of derivative rules we want to be able to take the derivative of any function. Currently with power rule and product/quotient rule any polynomial or rational function we can find the derivative of easy. But that still leaves trig functions, exponential functions and their inverse functions that we need to unpack.

Given the graph of sine, graph the derivative, hypothesize the function of the derivative, then prove that is true using limits.



Proof that

$$\frac{d}{dx} \sin x = \cos x$$

$$\lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h}$$

$$\lim_{h \rightarrow 0} \frac{\sin x \cos h + \sin h \cos x - \sin x}{h}$$

$$\lim_{h \rightarrow 0} \frac{\sin h}{h} \cos x + \lim_{h \rightarrow 0} \frac{\sin x (\cos h - 1)}{h}$$

$$= \underline{\underline{\cos x}}$$

$$\frac{d}{dx} \cos x = -\sin x$$

Once we know the derivative of sine and cosine the 4 other trig derivatives follow because they are built with sine and cosine.

Function, $f(x)$	$f(x) = g(\sin x, \cos x)$	Derivative, $f'(x)$
$\tan x$	$\frac{\sin x}{\cos x} = f$ $\cos x = g$	$\frac{f'g - g'f}{g^2} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \underline{\underline{\sec^2 x}}$
$\sec x$		
$\csc x$	$\frac{1}{\sin x} = f$ $\sin x = g$	$\frac{0 - \cos x}{\sin^2 x} = -\csc x \cot x$
$\cot x$	$\frac{\cos x}{\sin x} = f$ $\sin x = g$	$\frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = -\csc^2 x$

Practice Problems: 3.5: # 1-10, 14, 18, 20, 26, 27, 36



# 25, 30