Applied Rates of Change – More Practice

Goal: • Understands how to relate variables to time and to each other when taking differential equations • Can use geometric equations and understands their applications • Can use scientific equations involving a differential equation
Terminology: • Differential Equation

Volume: The volume V of a cone is related to the radius r and height h and can be expressed as

$$V = \frac{1}{3}\pi r^2 h$$

a. How is dV/dt related to dr/dt if h is constant? Write a sentence to describe a scenario where this would occur and draw a picture that models it.

b. How does the change in volume with respect to time relate to the change in radius and height if neither are constant? Write a sentence to describe a scenario where this would occur and draw a picture that models it.

c. How is the change in volume with respect to height related to the change in radius if neither r or h are constant? Write a sentence to describe a scenario where this would occur and draw a picture that models it.

Distance: Let x and y be the horizontal and vertical distance between two points. Then the distance between then points is

$$h = \sqrt{x^2 + y^2}$$

a. How is dh/dt related to dx/dt if y is constant? Write a sentence to describe a scenario where this would occur and draw a picture that models it.

b. How is the change in x with respect to time related to the change in y if h is constant? Write a sentence to describe a scenario where this would occur and draw a picture that models it.

c. How is the change in h related to the change in x and y over time if neither are constant? Write a sentence to describe a scenario where this would occur and draw a picture that models it.

d. How would this equation be extended for 3 dimensions?

Volume of Prism: The volume of a prism is related to the area of the base A and the height h as

$$V = A \cdot h$$

a. How is dV/dt related to dh/dt if A is constant? Write a sentence to describe a scenario where this would occur and draw a picture that models it.

b. How is the change in volume related to the change in height and area over time if neither are constant? Write a sentence to describe a scenario where this would occur and draw a picture that models it.

c. How is dA/dh related to dV/dt if neither are constant? Write a sentence to describe a scenario where this would occur and draw a picture that models it.

Economics: The cost to produce n units is $C(n)$ and the revenue is $R(n)$.

a. Determine an expression for the profit to produce n units, $P(n)$

b. How does dP/dn relate to dC/dn and dR/dn? Write a sentence to describe a scenario where dC/dn is constant but dR/dn is variable.

Physics: The kinetic energy of a moving object is related to its velocity and mass

$$K = \frac{1}{2}mv^2$$

a. How does dK/dt relate to dv/dt if mass is constant? Write a sentence to describe a scenario where this would occur and draw a picture that models it.

b. The force is related to mass and acceleration of an object

$$F = ma$$

How does F relate to the change in kinetic energy with respect to time?

c. Work is related to force and distance

$$W = Fd$$

How does W relate to the change in kinetic energy with respect to time? Interpret the results by considering $\Delta K/\Delta t$

Chemistry: For an ideal gas pressure, volume and temperature are related by

$$PV = nRT$$

Where n and R are constants.

a. How does the volume change with respect to temperature if pressure is constant?

b. For an ideal gas, the kinetic energy of a closed system is related to the heat added q and the energy lost by the gas changing volume.

$$E = q - PV$$

Relate the change in kinetic energy with respect to the change in volume. Draw a picture to illustrate heating a closed container of helium, such that the volume can change.

c. For monotonic ideal gasses (like helium), the kinetic energy is related to the change in its temperature as

$$E = \frac{3}{2}nRT$$

Relate the change in kinetic energy with respect to the change in the temperature of the gas.

d. The specific heat capacity of a gas is how much heat (Δq) is needed to change the temperature of the gas (ΔT). Show that the specific heat capacity at constant pressure is

$$\frac{dq}{dT} = \frac{5}{2}nR$$

e. Recall that the kinetic energy of a gas only depends on change in heat added and change in volume (not the change in pressure). Deduce that the specific heat capacity at constant volume is $\frac{3}{2}nR$