Related Rates

Goal: • Can model problems involving volume and surface area.
Terminology: • None
Reminder: • Quiz next week on Dec 13^{th} on Related Rates • Test at the end of the month Dec 18^{th}

We are going to continue practicing related rates for the next two days

Review: A person, who is 1.8 m tall, is walking 1 m/s down a street at night toward a lightpost that stands 7 m tall. How fast is the length of the person's shadow changing when the person is 5 m away from the lightpost?

Example: Water is flowing out of a conical fiter into a cylindrical cup at a constant rate of 1 cm^3/sec. If the filer has a height of 8 cm and diameter of 6 cm and the cylindrical cup has the same diamter determine how fast the height of water in the filter and cup are changing.

Practice Problems: 3.5: 2D # 1-3, 5, 8-15
3D # 4, 6, 7, 16, 17

Additional Related Rates Practice Problems

1. A cylindrical tank standing upright (with one circular base on the ground) has radius 20 cm. How fast does the water level in the tank drop when the water is being drained at 25 cm^3/sec?

2. A ladder 13 meters long rests on horizontal ground and leans against a vertical wall. The top of the ladder is being pulled up the wall at 0.1 meters per second. How fast is the foot of the ladder approaching the wall when the foot of the ladder is 5 m from the wall?

3. A point moves along the parabola $y = \frac{1}{4}x^2$ in such a way that at $x = 2$ the x-coordinate is increasing at the rate of 5 cm/s. Find the rate of change of y at this instant.

4. Sand is poured onto a surface at 15 cm3/sec, forming a conical pile whose base diameter is always equal to its altitude. How fast is the altitude of the pile increasing when the pile is 3 cm high?

5. A boat is pulled into a dock by a rope with one end attached to the front of the boat and the other end passing through a ring attached to the dock at a point 5 ft higher than the front of the boat. The rope is being pulled through the ring at the rate of 0.6 ft/sec. How fast is the boat approaching the dock when 13 ft of rope are out?

6. A balloon is at a height of 50 meters, and is rising at the constant rate of 5 m/sec. A bicyclist passes beneath it, traveling in a straight line at the constant speed of 10 m/sec. How fast is the distance between the bicyclist and the balloon increasing 2 seconds later?

7. A pyramid-shaped vat has square cross-section and stands on its tip. The dimensions at the top are 2 m by 2 m, and the depth is 5 m. If water is flowing into the vat at 3 m^3/min, how fast is the water level rising when the depth of water (at the deepest point) is 4 m? Note: the volume of any "conical'' shape (including pyramids) is $V = \frac{1}{3} height \times Area_{base}$

8. Water is leaking out of the bottom of an inverted conical tank at the rate of 0.1 m^3/min, and at the same time is being pumped in the top at a constant rate of k m^3/min. The tank has height 6 m and the radius at the top is 2 m. Determine the constant k if the water level is rising at the rate of 0.2 m/min when the height of the water is 2 m.

9. A woman 5 ft tall walks at the rate of 3.5 ft/sec away from a streetlight that is 12 ft above the ground. At what rate is her shadow lengthening? At what rate is the tip of her shadow moving?

10. A man 1.8 meters tall walks at the rate of 1 meter per second toward a streetlight that is 4 meters above the ground. At what rate is his shadow shortening? At what rate is the tip of his shadow moving?

11. A police helicopter is flying at 150 mph at a constant altitude of 0.5 mile above a straight road. The pilot uses radar to determine that an oncoming car is at exactly 1 mile from the helicopter, and that this distance is decreasing at 190 mph. Find the speed of the car.

12. A light shine from the top of a pole 20 m high. A ball is falling 10 meters from the pole, casting a shadow on a building 30 meters away. When the ball is 25 meters from the ground it is falling at 6 meters per second. How fast is its shadow moving?

13. A road running north to south crosses a road going east to west at the point P. A car is driving north along the first road, and a plane flying east above the the second road. At a particular time, the car is 10 kilometers to the north of P and traveling at 80 km/hr, while the plane is 15 kilometers to the east of P at an altitude of 2 km and traveling at 200 km/hr. How fast is the distance between the car and plane changing?

14. Same as number 11 above, but instead the plane is travelling west and gaining altitude at a rate of 10 km/h.

15. A light shines from the top of a pole 20 m high. An object is dropped from the same height from a point 10 m away, so that its height at time t seconds is $h(t) = 20 - 4.9t^2$. How fast is the object's shadow moving on the ground one second later?

16. Consider a cylindrical tree trunk of radius R. Living cells occupy a thin shell (thickness x) just inside the tree bark. The interior of the trunk consists of dead cells that have turned into wood. Let F be the fraction of the trunk volume that is living tissue. How does F change with time as the tree grows at the instant that the radius is 5 times the thickness x? Assume that the radius of the trunk grows at a constant rate, and that the thickness x does not change.

17. During early development, the limb of a fetus increases in size, but has constant proportions. Suppose that the limb is roughly a circular cylinder with radius r and length L in proportion
$$\frac{L}{r} = C$$
where C is a positive constant. It is noted that during the initial phase of growth, the radius increases at an approximately constant rate a. At what rate does the mass of the limb change during this time? Assume the density of the limb is 1 g/cm^3.

18. A convex lens has a focal length of $f = 10$ cm. Let p be the distance between an object and the lens, and q the distance between its image and the lens. These distances are related to the focal length f by the equation:
$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$
Consider an object which is 30 cm away from the lens and moving away at 4 cm/sec. How fast is its image moving and in which direction?

Solutions

1. $\frac{1}{16\pi} = 0.0199$ cm/s
2. $\frac{6}{25} = 0.24$ m/s
3. 5 cm/s
4. $\frac{20}{3\pi} = 2.12$ cm/s
5. $\frac{13}{20} = 0.65$ ft/s
6. $\frac{5\sqrt{10}}{2} = 7.91$ m/s
7. $\frac{75}{64} = 1.17$ m/min
8. $k = 0.1 + \frac{4\pi}{45}$ m³/min
9. Length: 2.5 ft/s; Tip: 6 ft/s
10. Length: $\frac{9}{11} = 0.818$ m/s; Tip: $\frac{20}{11} = 1.82$ m/s
11. $\frac{380}{\sqrt{3}} - 150 = 69.4$ mph
12. 18 m/s
13. $\frac{3800}{\sqrt{329}} = 210$ km/h
14. $-\frac{2180}{\sqrt{329}} = -120$ km/h
15. $\frac{4000}{49} = 81.6$ m/s
16. $-\frac{8}{125x} \cdot \frac{dR}{dt}$
17. $3\pi \cdot arL$
18. 1 cm/s toward lens