

## Area Approximation Practice Solutions

1.  $f(x) = x^2$  on  $[0, 2]$  using 4 subintervals

Here  $x_k = \frac{k}{2}$  and  $m_k = x_k + \frac{1}{2}\Delta x = x_k + 0.25$

Type	Sigma Notation	Value
Right Endpoint	$\frac{1}{2} \sum_{k=1}^4 x_k^2$	3.75
Left Endpoint	$\frac{1}{2} \sum_{k=0}^3 x_k^2$	1.75
Midpoint	$\frac{1}{2} \sum_{k=0}^3 m_k^2$	2.625
Trapezoid	$\frac{1}{2} \left( 0^2 + 2 \sum_{k=1}^3 x_k^2 + 2^2 \right)$	2.75
Right Endpoint (Limit)	$\lim_{n \rightarrow \infty} \sum_{k=1}^n \left( \frac{2k}{n} \right)^2 \cdot \frac{2}{n}$	2.666 ...

2.  $f(x) = \ln x$  on  $[1,3]$  using 8 subintervals

Here  $x_k = 1 + k\Delta x = 1 + \frac{k}{4}$ , and  $m_k = x_k + \frac{1}{2}\Delta x = x_k + \frac{1}{8}$

Type	Sigma Notation	Value
Right Endpoint	$\frac{1}{4} \sum_{k=1}^8 \ln x_k$	1.429..
Left Endpoint	$\frac{1}{4} \sum_{k=0}^7 \ln x_k$	1.155..
Midpoint	$\frac{1}{4} \sum_{k=0}^7 \ln m_k$	1.297..
Trapezoid	$\frac{1}{2} \left( \ln 1 + 2 \sum_{k=1}^7 \ln x_k + \ln 3 \right)$	1.292..
Right Endpoint (Limit)	$\lim_{n \rightarrow \infty} \sum_{k=1}^n \ln \left( 1 + \frac{2k}{n} \right) \cdot \frac{2}{n}$	1.2958..

3.  $f(x) = \frac{1}{x^2+1}$  on  $[-3, 3]$  using 6 subintervals.

Here  $x_k = -3 + k\Delta x = -3 + k$ , and  $m_k = x_k + \frac{1}{2}\Delta x = x_k + \frac{1}{2}$

Type	Sigma Notation	Value
Right Endpoint	$\sum_{k=1}^6 \frac{1}{x_k^2 + 1}$	2.5
Left Endpoint	$\sum_{k=0}^5 \frac{1}{x_k^2 + 1}$	2.5
Midpoint	$\sum_{k=0}^5 \frac{1}{m_k^2 + 1}$	2.491..
Trapezoid	$\frac{1}{2} \left( \frac{1}{10} + 2 \sum_{k=1}^5 \frac{1}{x_k^2 + 1} + \frac{1}{10} \right)$	2.5
Right Endpoint (Limit)	$\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{\left(-3 + \frac{6k}{n}\right)^2 + 1} \cdot \frac{6}{n}$	2.4980..